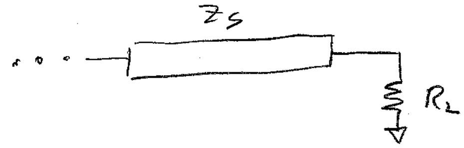
① $\Gamma_L = \dfrac{Z_L - Z_S}{Z_L + Z_S}$, $Z_S = 50\Omega$



a) $Z_L = 10\Omega$, $\Gamma_L = -0.67$

b) $Z_L = 50\Omega$, $\Gamma_L = 0$

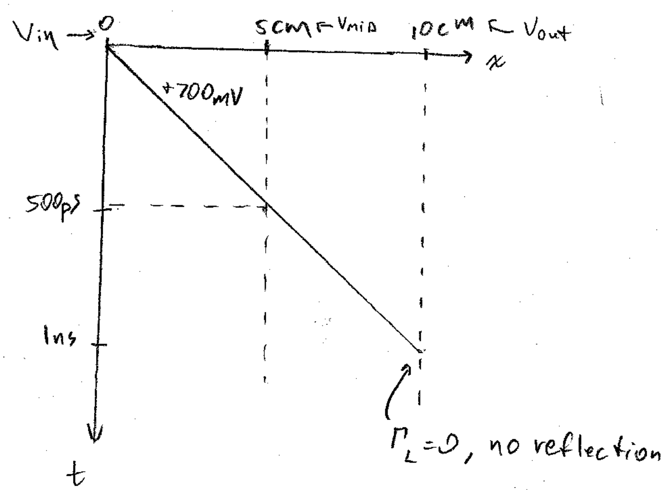
c) $Z_L = 200\Omega$, $\Gamma_L = 0.6$

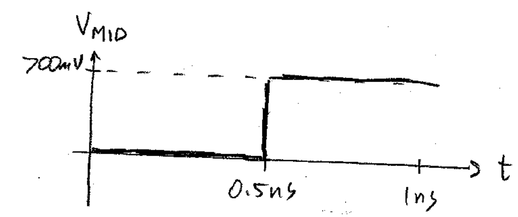
② case (A/B)

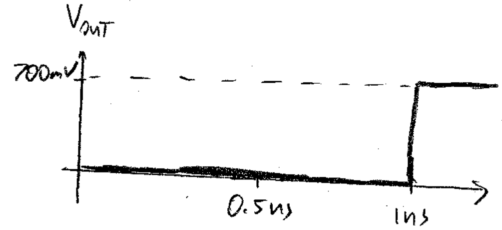
$Z_L = Z_0 \rightarrow \Gamma_L = 0$

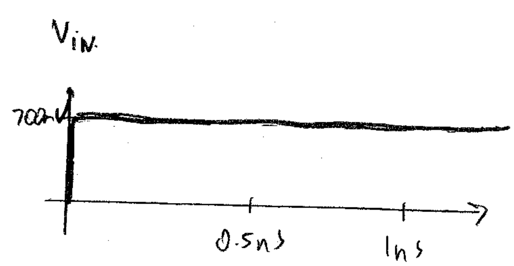
$\tau = 10cm / 10^8 m/s = 1ns$

$\tau_{mid} = 5cm / 10^8 m/s = 0.5ns$







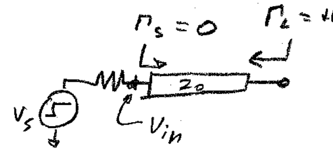


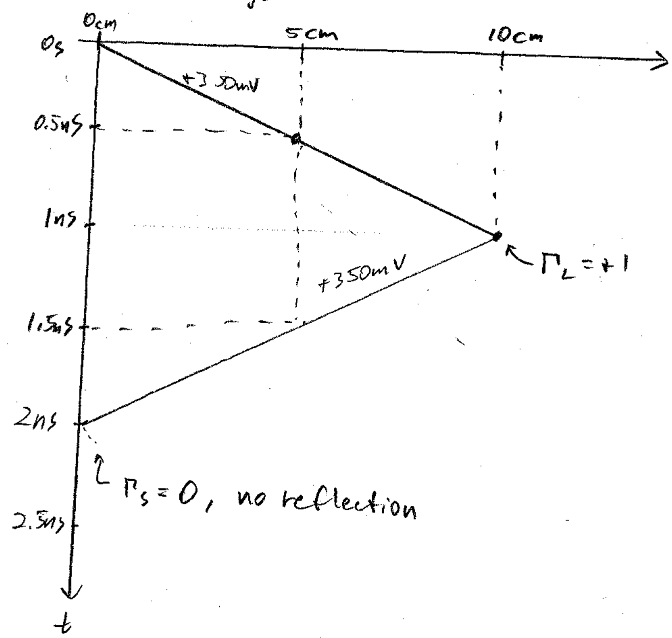
CASE C/D

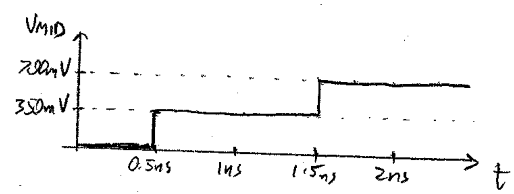
$Z_L = \infty \rightarrow \Gamma_L = +1$

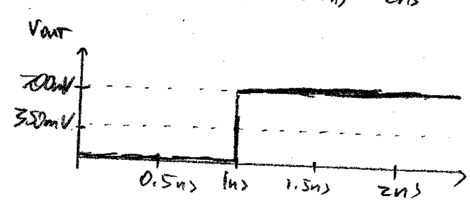
$Z_S = Z_0 \rightarrow \Gamma_S = 0$

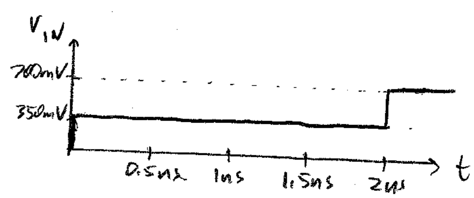
$V_{in} = \dfrac{Z_0}{Z_0 + R_{gen}} = \frac{1}{2} V_S = 350mV$







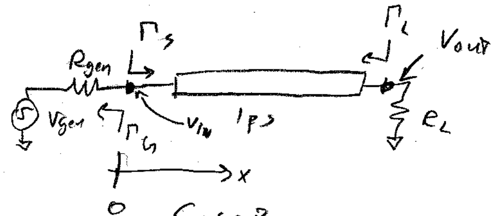




(2) -continued-

Comments: For driving a digital-bus, it is generally desired to have a single rising edge of the maximum magnitude. This favors load-side termination (Case A/B). With source-side termination V_{MID} has two rising edges and takes longer to reach full magnitude.

(3) $Z_0 = 100\Omega$, $\tau = 1ps$, $V_{gen} = 1V$ step



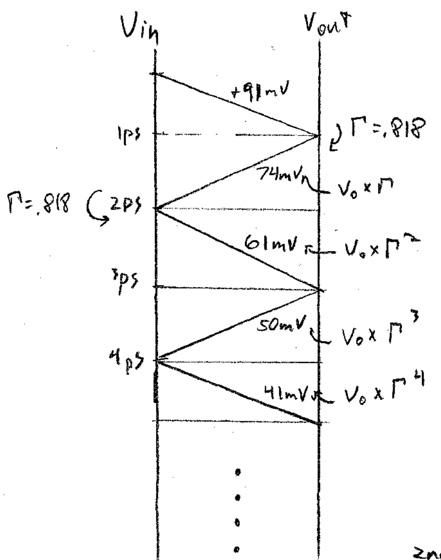
Case A

$R_L = R_{gen} = 1k\Omega$

$\Gamma_L = \Gamma_S = 0.818$

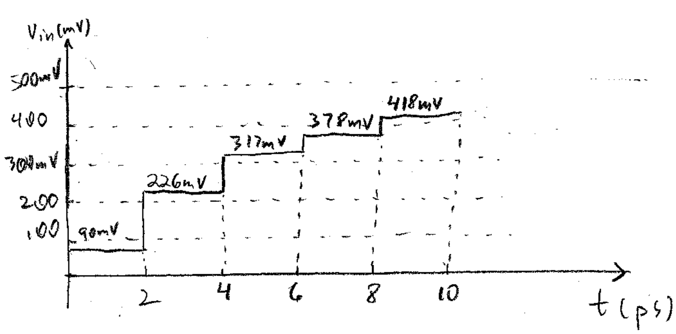
$\Gamma_G = -0.818 \rightarrow V_{in}(t=0) = \frac{Z_0}{Z_0 + R_g}$

$V_{in}(t=0) = 91mV = V_0$

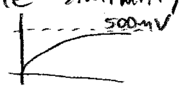


$$V_{in} = V_0 (\underset{\text{initial}}{1} + \underset{\text{1st step}}{(\Gamma + \Gamma^2)} + \underset{\text{2nd step}}{(\Gamma^3 + \Gamma^4)} + \underset{\text{3rd step}}{(\Gamma^5 + \Gamma^6)} + \dots)$$

$V_{in}(t=\infty) = 500mV$ from Voltage Divider



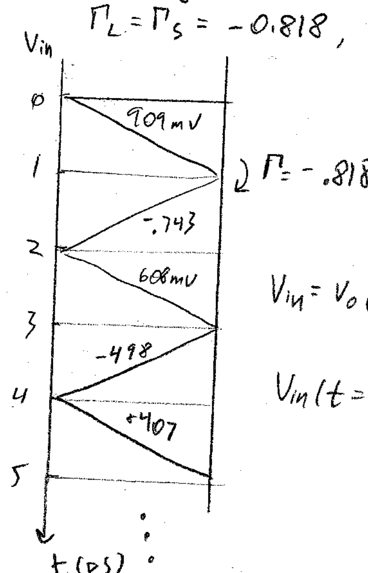
Notice similarity to exponential function



Case B

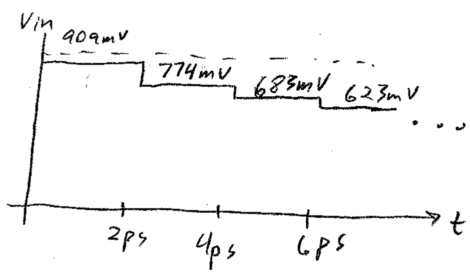
$R_L = R_{gen} = 10\Omega$

$\Gamma_L = \Gamma_S = -0.818$, $V_{in}(t=0) = \frac{100}{100+10} = 909mV$

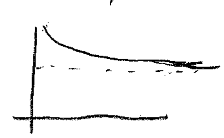


$V_{in} = V_0 (1 + \Gamma + \Gamma^2 + \Gamma^3 + \Gamma^4 + \dots)$

$V_{in}(t=\infty) = 500mV$

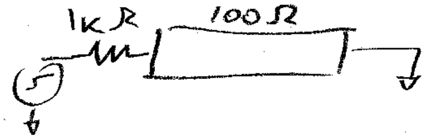


Notice similarity to exponential:



③ -cont.-

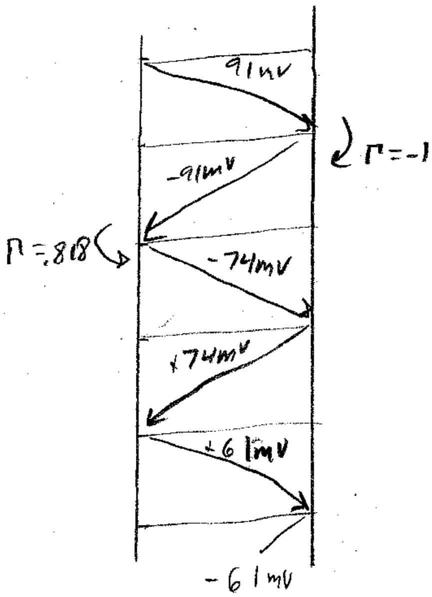
Case C

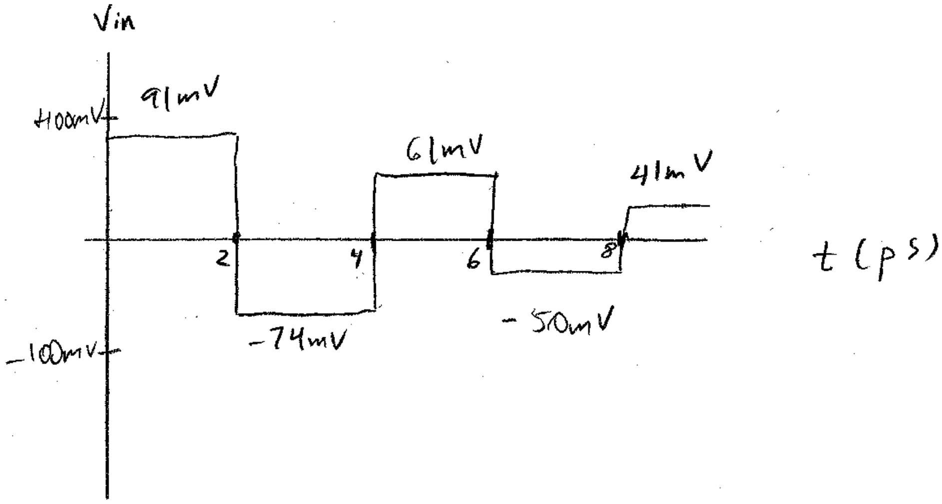


$R_L = 0\Omega$, $R_g = 1k\Omega$

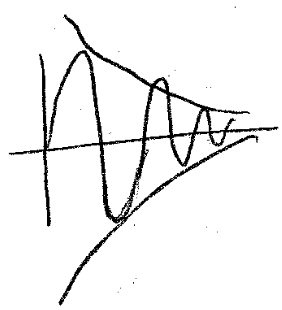
$\Gamma_L = -1$ $\Gamma_S = +0.818$

$V_0 = 91mV$, as before





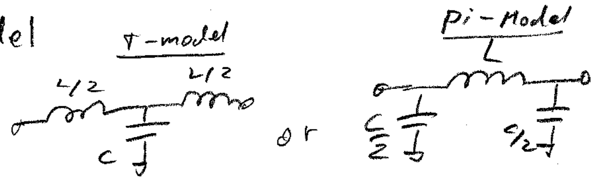
Note similarity to exponentially dying oscillation



(4) Lumped element model

$L_T = \tau Z_0$

$C_T = \tau / Z_0$

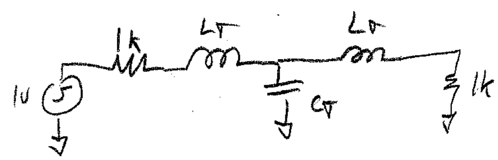


$Z_0 = 100\,\Omega,\ \tau = 1ps$

So: $L_T = 100pH$

$C_T = 10\ fF$

a)

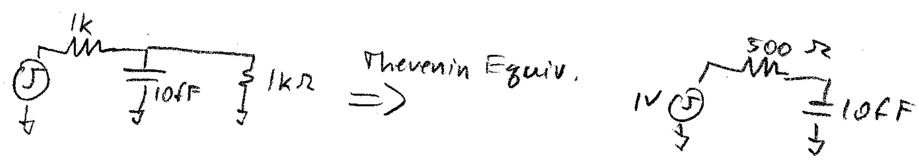


$R_L = R_g = 1k\Omega$

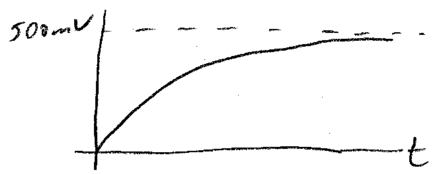
$\tau_{RL} = \frac{50pH}{1k\Omega} = 50fs \qquad \tau_{RC} = 10fF \times \underbrace{1k\Omega \,\|\, 1k\Omega}_{500\Omega} = 5ps$

$\tau_{RC} = 100 \times \tau_{RL}$

$\tau_{RL} \ll \tau_{RC} \Rightarrow$ Discount inductor effects



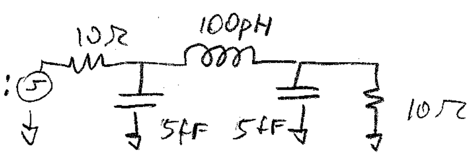
$V_{in} = (1 - e^{-t/\tau_{RC}})V_{gen} = \boxed{0.5(1-e^{-t/5\times10^{-12}})\ [V]}$



↳ Note similarity to prob. 3 a

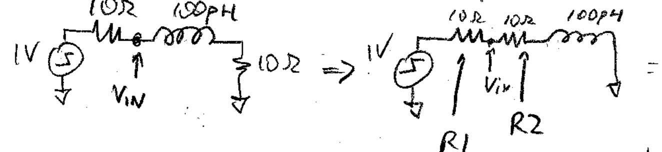
b) $R_L = R_{gen} = 10\Omega$

Use T-model:



$\tau_{RL} = \frac{100pH}{20\Omega} = 5ps \qquad \tau_{RC} = 5fF \times 10\Omega = 50fs$

$\tau_{RC} \ll \tau_{RL}$ so discount capacitor



$i(t) = V_{gen}/R\,(1-e^{t/\tau})$

$i(t) = 50mA \times (1-e^{t/5ps})$

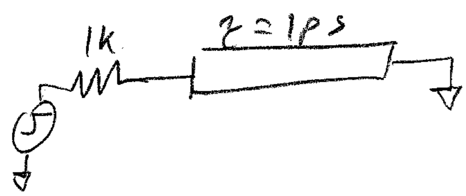
$V_{IN} = V_{R1} = 1V - 10\Omega \times i(t)$

$\boxed{V_{in} = 1 - 0.5e^{-t/5ps}\ [V]}$

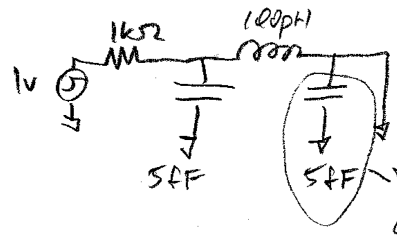
↳ Note similarity to prob 3 B

(4) - cont. -

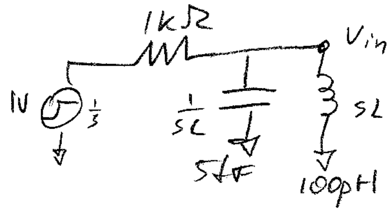
) $R_L = 0\ \Omega$, $R_{gen} = 1k\Omega$



Use Pi-Model:



This capacitor is grounded, remove it



We have a resonator

Nodal analysis in Laplace domain

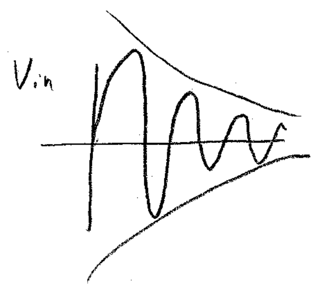
$$V_{in} sC + \frac{V_{in}}{sL} + \frac{V_{in} - V_{gen}/s}{R} = 0$$

$$V_{in} s^2 LC + V_{in} + \frac{V_{in} s L}{R} = \frac{V_{gen} L}{R}$$

$$\boxed{V_{in} = \frac{V_{gen} L/R}{s^2 LC + s\frac{L}{R} + 1}} = \frac{A}{\frac{s^2}{\omega_n^2} + \frac{2\zeta}{\omega_n} s + 1}$$

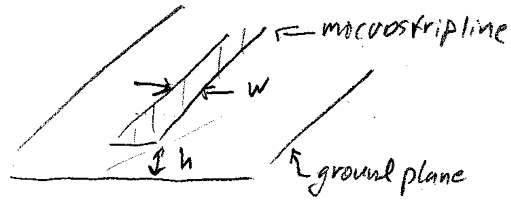
$$\omega_n = 1/\sqrt{LC}, \quad \zeta = \frac{1}{2R}\sqrt{\frac{L}{C}} = .07$$

$\rightarrow$ very underdamped



Note similarity to problem 3C

(5) $l = 300\mu m$, $V_p = c$, $Z_0 = 100\Omega$



since $V_p = c/\sqrt{\varepsilon_R}$, $\varepsilon_r = 1$, $\tau = \frac{l}{V_p} = 1ps$

Pozar Eqn 4.206 pp.185: (this can be found from Eqn 2.5.4 in Gonzalez)

$$\frac{W}{h} \simeq \frac{8e^{A}}{e^{2A}-2} \quad \text{for } \frac{W}{h} < 2 \quad \text{and } A = \frac{Z_0}{60}\sqrt{\frac{\varepsilon_r+1}{2}} + \frac{\varepsilon_r-1}{\varepsilon_r+1}\left(.23 + \frac{.11}{\varepsilon_R}\right)$$

$$= 5/3$$

$\boxed{\frac{W}{h} = 1.63}$ (if $\frac{W}{h} < 2$ is a good assumption)

so if we assume $h = 30\mu m$ then $W = 49\mu m$

$L_T = \tau Z_0 = 1ps \times 100\Omega = 0.1nH$

$f_{max} = \frac{1}{\tau} = 1\,THz$, the freq at which $\lambda = l$. see plot.

(6) $C_T = \tau / Z_0 = 10fF$. See Plot.

Conclusion: Trasmission lines can be used to approximate ideal inductors and capacitors, but is less accurate at high frequencies. One application is to use a long skinny μstrip line which acts like an inductor to provide DC-bias instead of using an RF choke. You will use this in Lab 3.