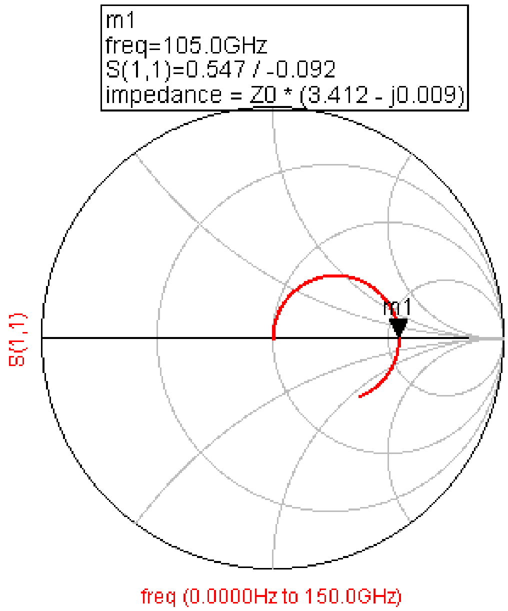
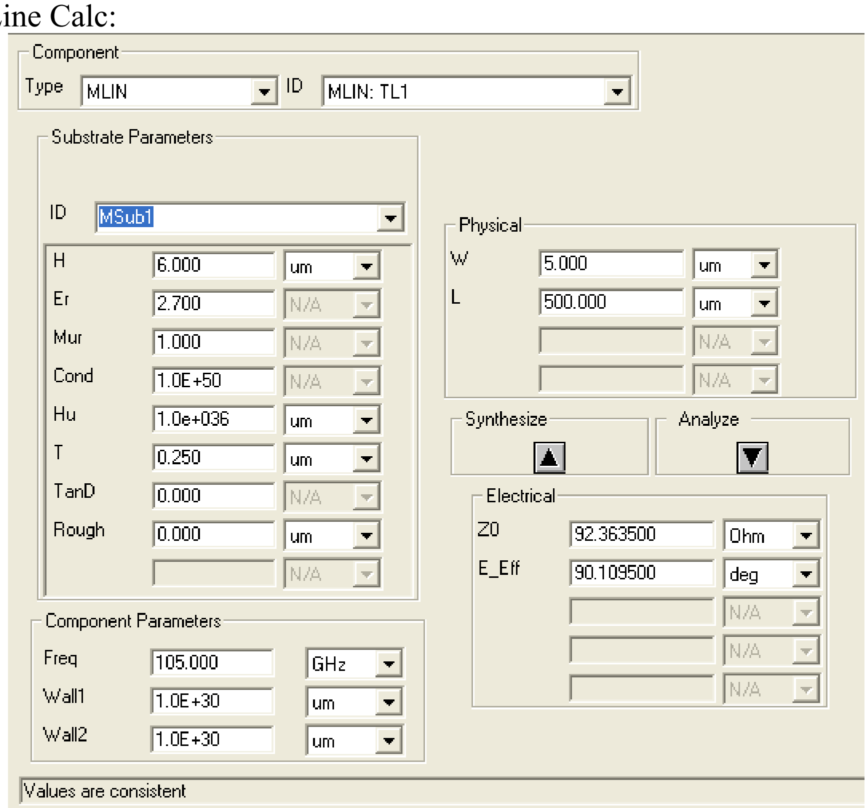
ECE 145A/218A W09 HW3 Solutions

Problem #1

Recall that a quarter wave transformer will transform any real impedance into another real impedance according to: $Z_{IN}(\lambda/4) = (Z_0)^2 / Z_L$. Therefore when the frequency reaches a value such that the line length is an odd-multiple of a quarter-wavelength, the input impedance will cross the real axis with the impedance described in the preceding equation. In this case the load impedance is the 50Ω termination of port 2.

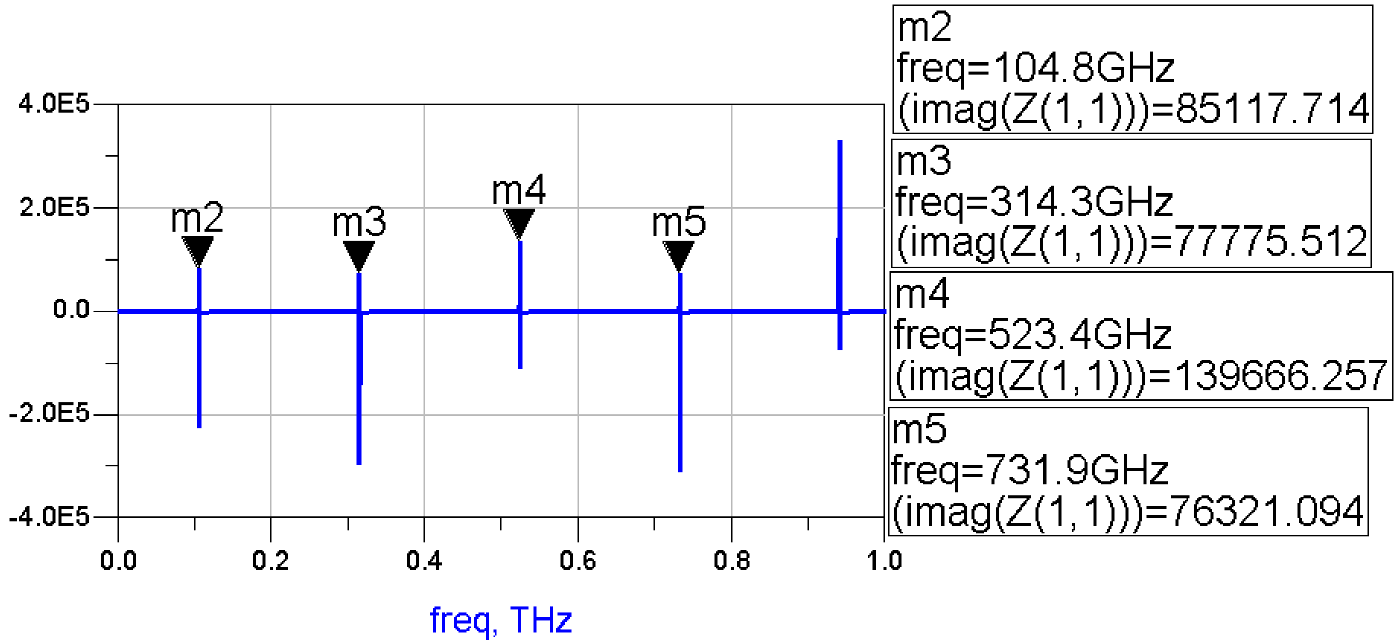


We see above that for $L=\lambda/4$, $Z_{IN} = 3.4 + j*0$.
Therefore $Z_0 = \text{sqrt}(Z_{IN} * Z_L) = 50 * \text{sqrt}(3.412 * 1) =$ 92.4Ω
This is confirmed in Line Calc:



To determine the propagation velocity v_P, we use the fact that for a short-circuited transmission line $Z_{SC} = j*Z_0*\tan(\beta d)$, where $\beta = 2\pi/\lambda = 2\pi f / v_P$. Since tangent goes to infinity for arguments equal to odd multiples of $\pi/2$, or where d is an odd multiple of $\lambda/4$, and we see below that there are peaks at odd multiples of roughly 105GHz, at that frequency 500µm must be $\lambda/4$.

So, $v_P = f*\lambda = 105*10^9 * (4*500\mu m)$ = <u>2.1*10^8 m/s</u>
Check with $\varepsilon_{eff} = 2.142$ from Line Calc, v_P = 3*10^8/sqrt(2.142) = 2.1*10^8 m/s.



Problem #2

Part A

Width constraint: keep width below $\lambda/2$ at all design frequencies to prevent resonances. $\lambda = c/(f*sqrt(\varepsilon_R)) =$ 609um. W_{max} = 304.5um. Note: $\varepsilon_{effective}$ can be significantly less than ε_R, but because the line is going to be very wide the two will be very close. For narrow lines $\varepsilon_{Effective}$ will approach 1.

Length constraint: Keep impedance below infinity over design frequency range. Since impedance is infinite at DC (open circuit), quarter-wave will bring us to short circuit, and half-wavelength back to open circuit. So $L_{max} = \lambda/2$ = 304.5um.

Thus the low frequency capacitance $C = \varepsilon A/d$ = 371 fF

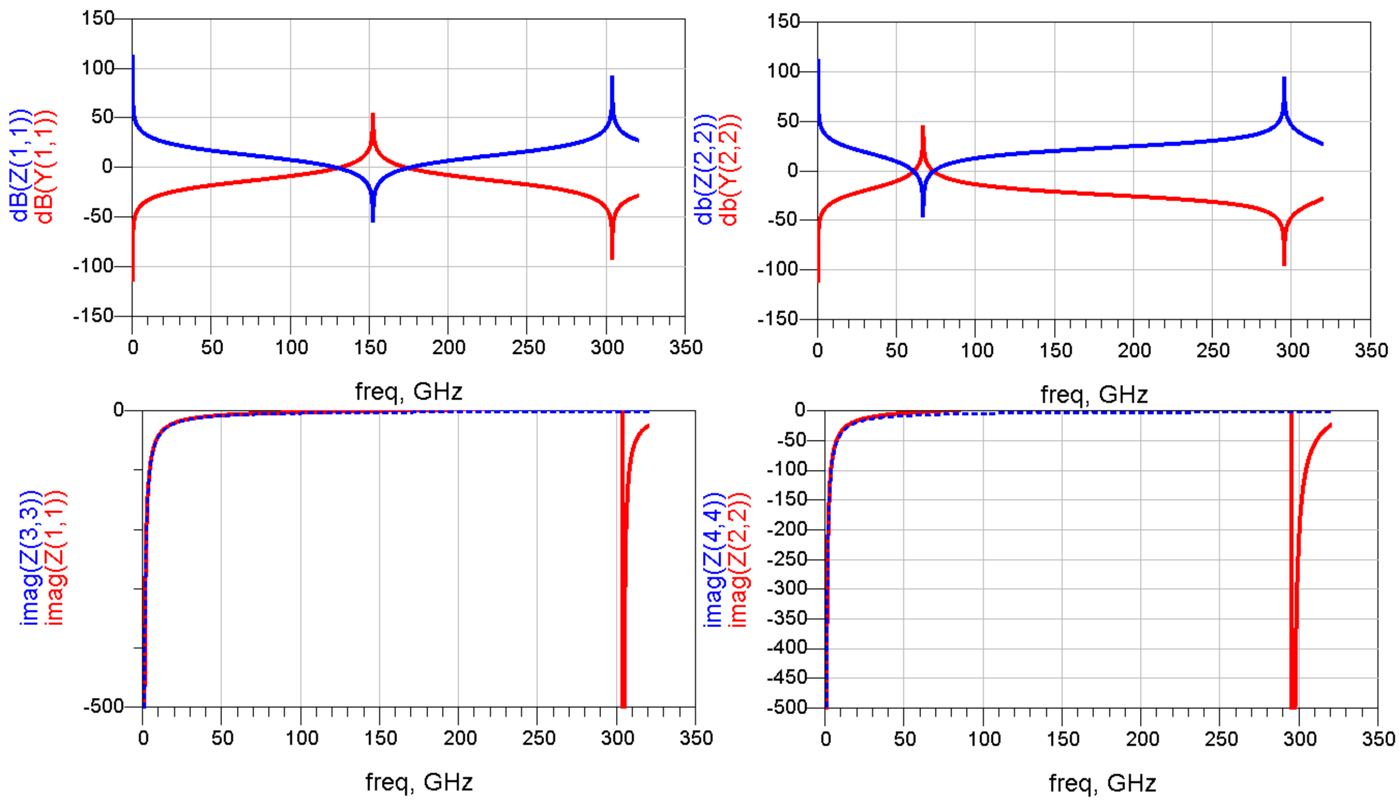
Part C

Because the radial stub is narrow, then widens, we must use $\varepsilon_{Effective}$ instead of just ε_R. But it changes over the entire length of the stub, since the relevance of the fringing effect depends on line width. As a quick, but reasonably accurate, approximation, take the average of the maximum: 2.7, and the minimum: 1, giving $\varepsilon_{Effective}$=1.85

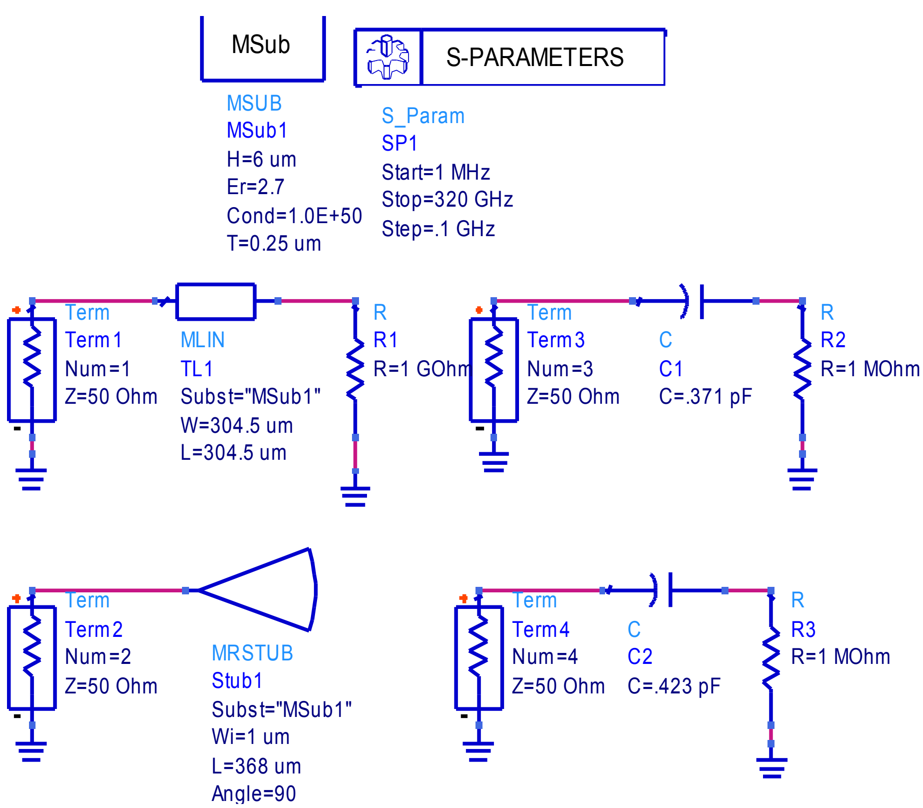
$L_{max} = \lambda/2 = c/(2*f*sqrt(\varepsilon_{Effective}))$ = 368um. $A = \pi*L^2/4$. Thus, C = 423fF.

Part B & D

Plots and schematic on next page. Z11 is rectangular line, Z22 is radial stub, Z33 is the capacitor from part A, Z44 is the capacitor from part C. We can see that with decent accuracy the impedance was kept finite up to 300GHz. The lower plots also show us that until the frequency increases such that the $L = \lambda/2$, for low frequencies there is close agreement between the imaginary components of impedance for the transmission line or stub, and the approximate capacitance calculated above. As frequency approaches the half-wavelength point, the discrepancy gets progressively larger.



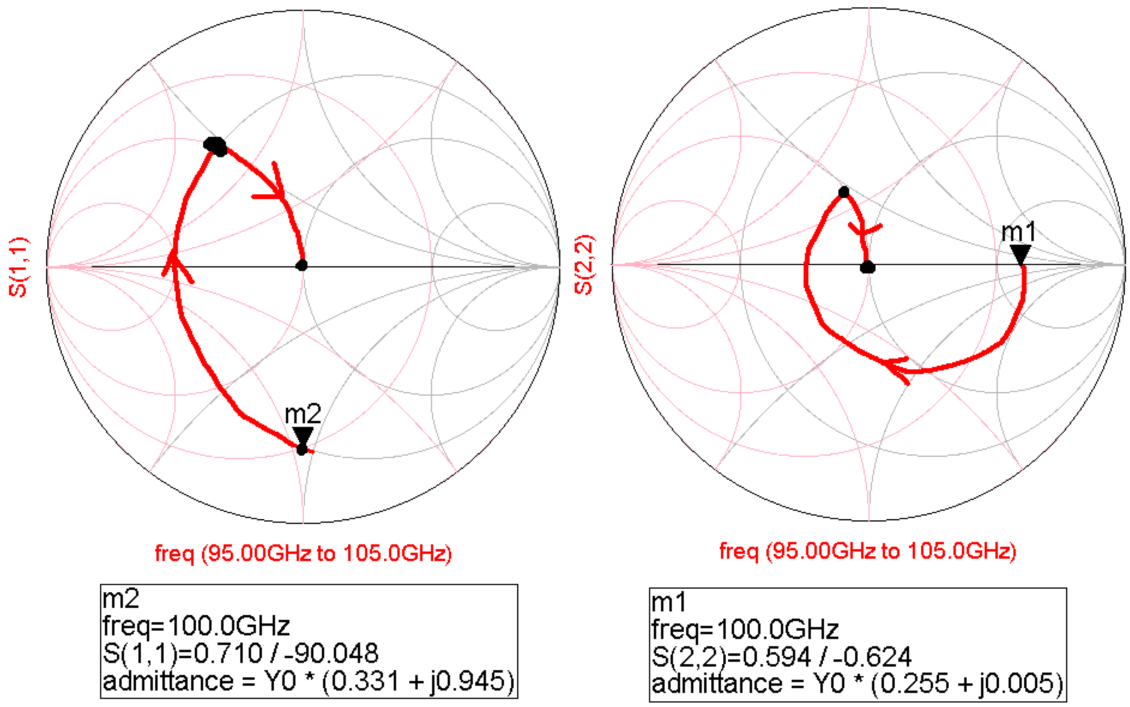
Top left: Rectangular Line Y and Z; **Top right:** Radial stub Y and Z; **Bottom left:** Rectangular line vs. capacitor; **Bottom right:** Radial stub versus capacitor

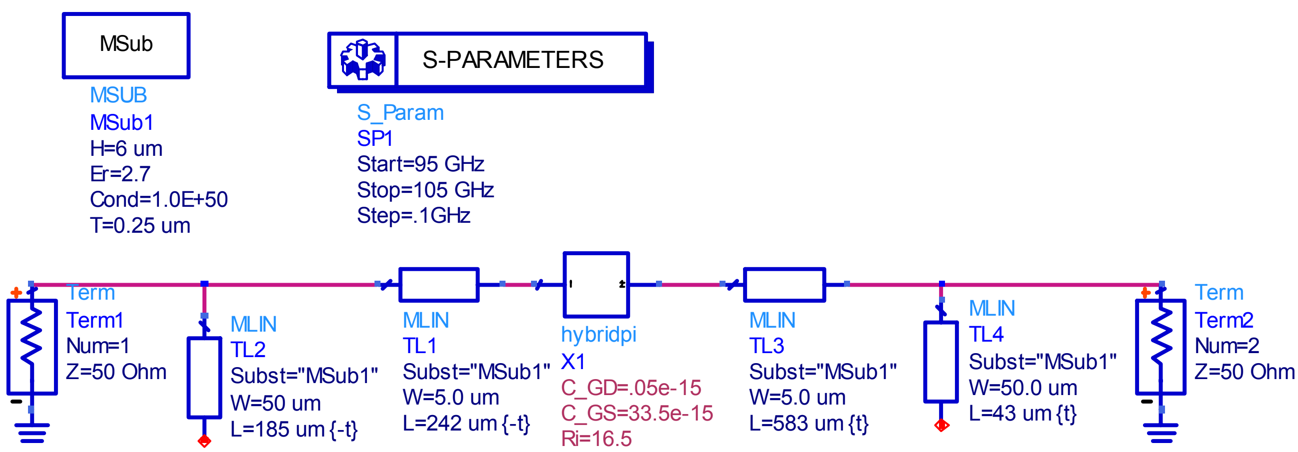


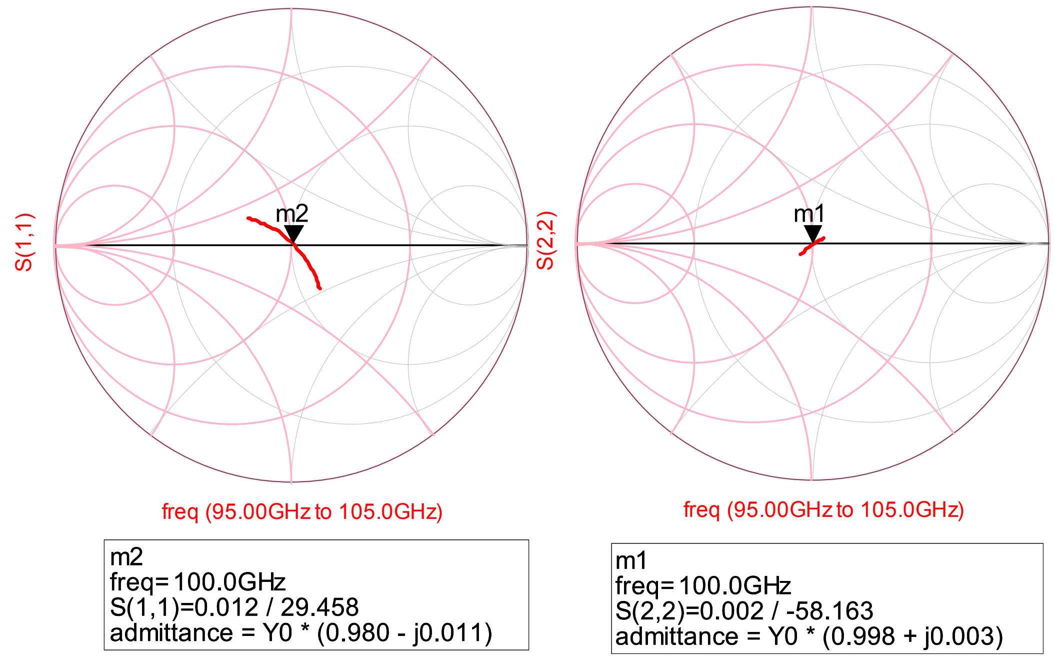
Problem #3

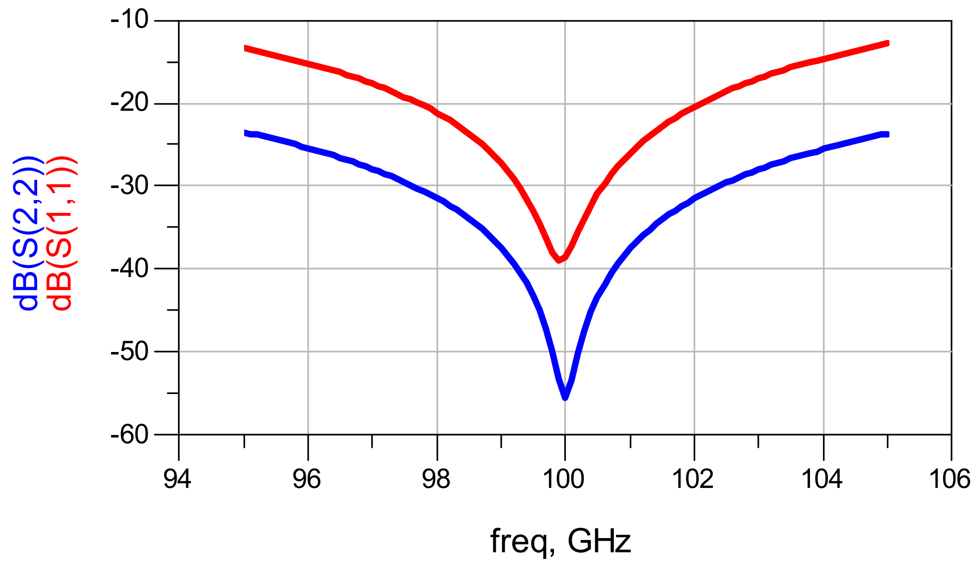
A) High- low-Z matching

This is a matching network using high and low impedance lines. Linecalc shows that the 50um wide line has Z_0=21.5Ω , and the 5um line has Z_0=92.3Ω. The high impedance lines behave more or less as an average between a 50Ω line an inductor, the low-impedance open stub behaves like a capacitor. Multiple solutions are possible, but here it has been designed at the input and output to use a series high-z line to bring the impedance to the unit admittance circle, then use an open stub to remove the remaining susceptance. See the diagram and final schematic below. We can see from the smith chart and dB plots of S11, S22, that the match is good.









B) Approximation of high-, low-z lines

Low-Z transmission lines can be approximated as a Pi-network, and the inductance $L=\tau*Z_0$ will be ignored because Z_0 is small. This leaves only a grounded shunt capacitor. For high-Z lines, the capacitance $C=\tau/Z_0$ will be ignored since Z_0 is large. This leaves a series inductor. The higher and lower the high- and low-z lines are respectively, the better approximation this becomes.

- $\tau = L/v_P$
- $v_P = c/\text{sqrt}(\varepsilon_R) = 3e8/\text{sqrt}(2.7) = 1.83E8$ m/s

High-Z lines:

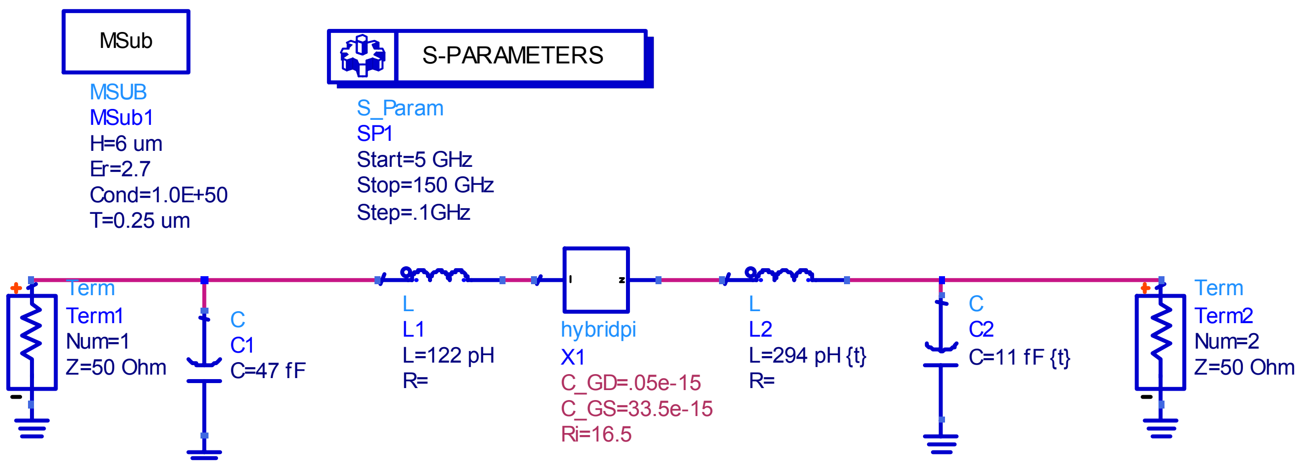
- Input:
  - $Z_0 = 92.3\Omega$ (LineCalc)
  - L=242um
  - τ=242um/1.83E8[m/s] = 1.3 ps
  - $L = \tau Z_0$ = 1.3ps * 92.3 = 122pH

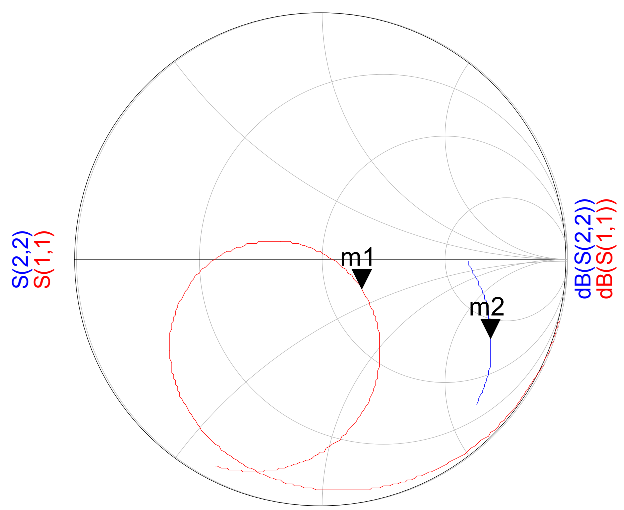
- Output
  - $Z_0 = 92.3\Omega$ (LineCalc)
  - L=583um
  - τ=583um/1.83E8[m/s] = 3.19 ps
  - $L = \tau Z_0$ = 3.19ps* 92.3 = 294pH

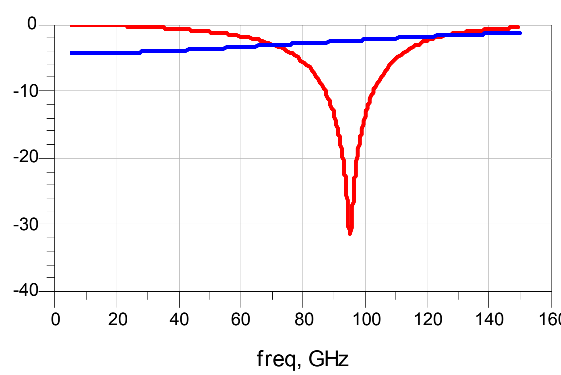
Low-Z Lines

- Input
  - $Z_0 = 21.5\Omega$
  - L=185um
  - T = 185um/1.83E8(m/s) = 1.01 ps
  - $C = \tau/Z_0$ = 1.01ps / 21.5Ω = 47fF
- Ouput
  - $Z_0 = 21.5\Omega$
  - L=43um
  - T = 185um/1.83E8(m/s) = 0.235 ps
  - $C = \tau/Z_0$ = 0.235ps / 21.5Ω =11 fF

Schematic and results:







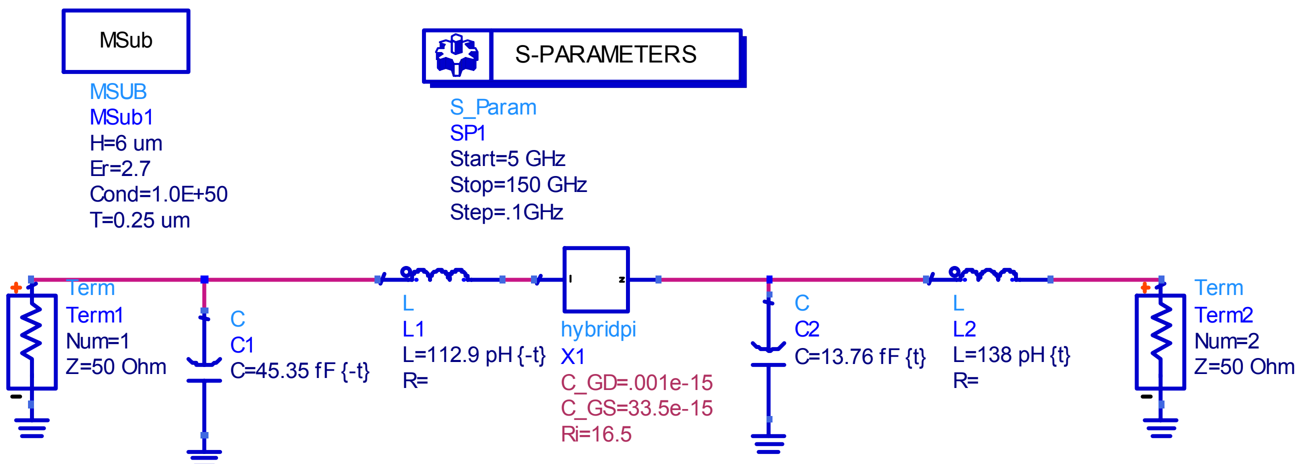
freq (5.000GHz to 150.0GHz)

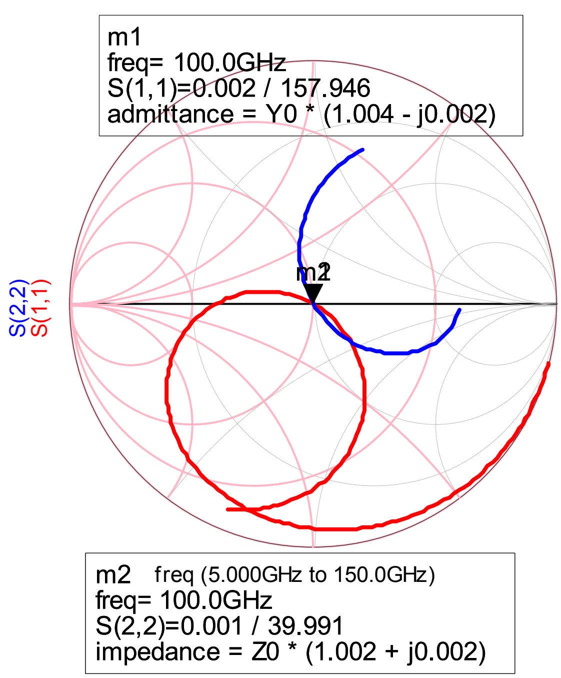
m1	m2
freq= 100.0GHz	freq= 100.0GHz
S(1,1)=0.205 / -35.780	S(2,2)=0.762 / -24.850
impedance = Z0 * (1.350 - j0.338)	impedance = Z0 * (2.122 - j3.236)

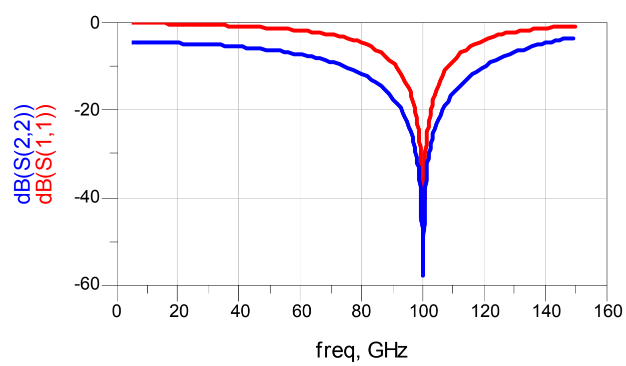
We can see in the smith and dB plots of the input and output reflection that the input is fairly well matched by the use of this approximate lumped element model. An input reflection of better than 30dB is achieved, but the matching frequency was shifted down almost 10GHz. The output reflection does not work well at all because of the differences between a real inductor and a high-z line. The high-z line was used to rotate about the origin to reach the unity admittance circle, but since the inductor only moves on circles of constant resistance, it cannot ever bring the output impedance to the left side of the smith chart.

C) LC Match

As stated above, the LC topology based on the transmission line match can never match the output to 50Ω. To make it possible, the position of the capacitor and inductor are swapped, so the shunt capacitor will move the output impedance to the unit impedance circle, then the inductor will rotate up this circle to 50Ω. Note that the proper LC values for the input match are quite close to the approximate values in (B). This shows that the approximate conversions between high- and low-z transmission lines as just inductors or capacitors respectively can work reasonably well, but can also fail if one does not properly recognize the differences between them.

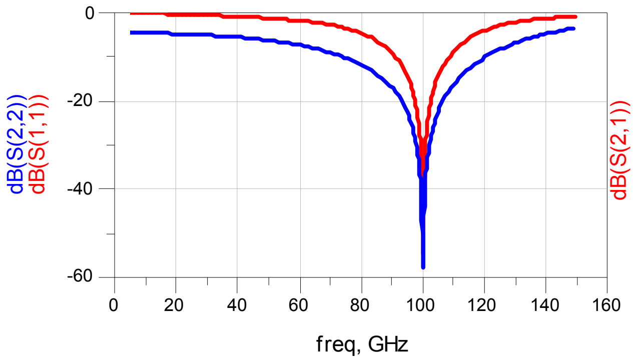


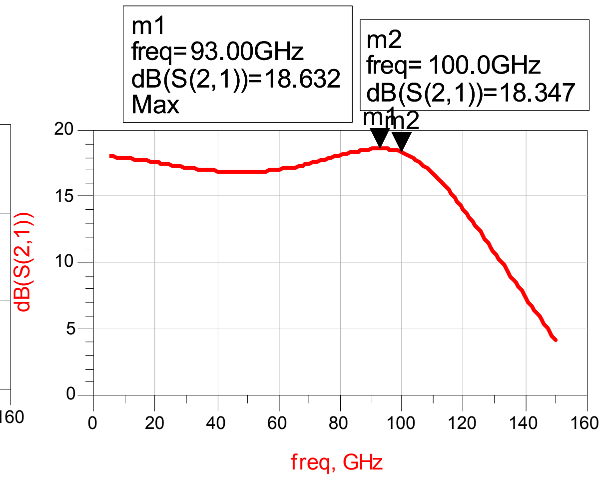




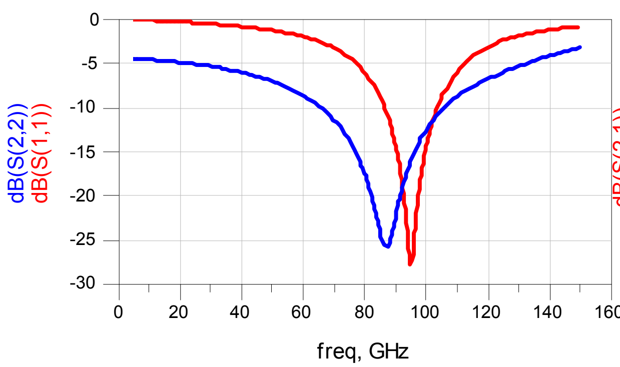
Problem #4

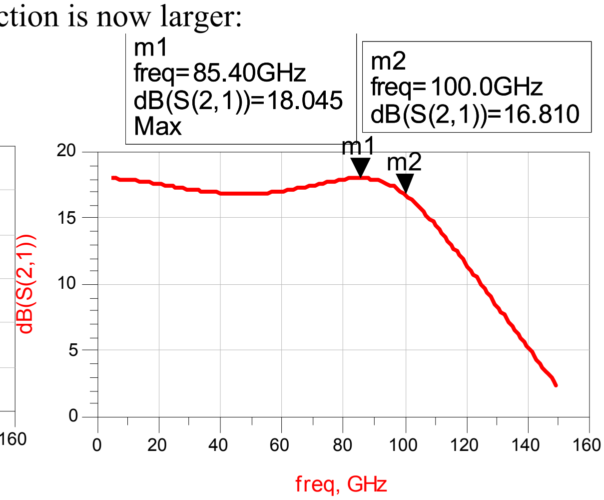
A,B,C)
Initially the model we are given is the same as in problem 3, so our LC matching network from 3c still works. Following are the dB plots showing an excellent match. Because of the small C_{GD}, after creating the output match, the input match did not need to be adjusted.



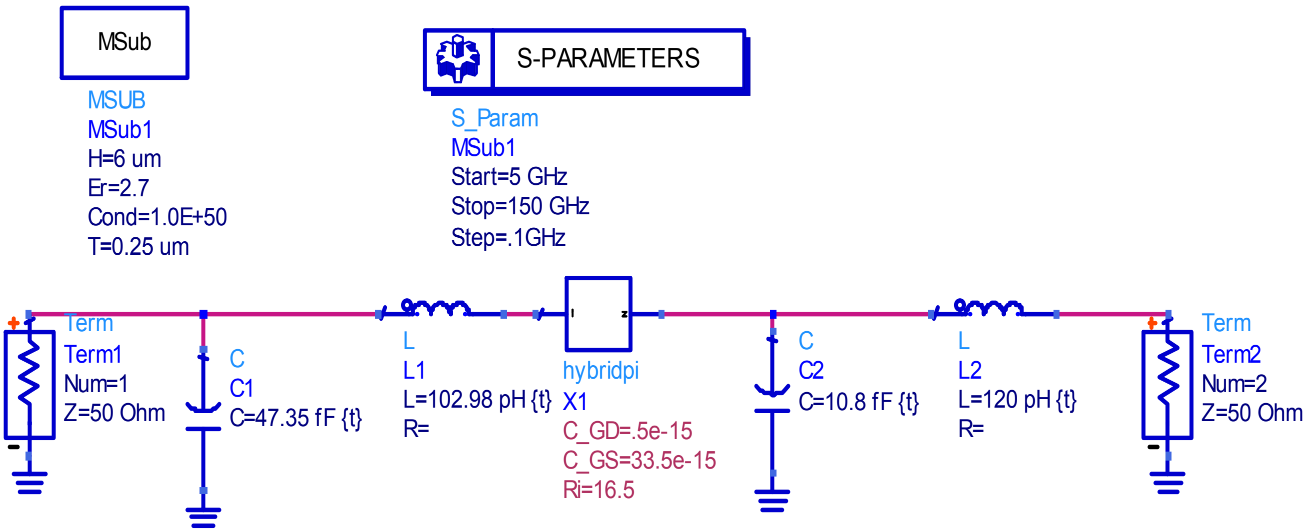


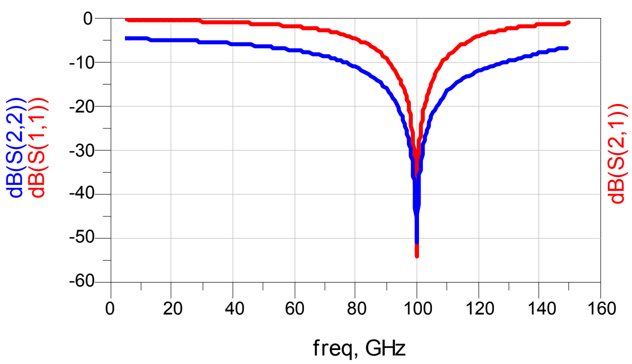
D)
C_{GD} is increased to 0.5fF, resulting in reduced peak and match-frequency gain. The intput and output reflection match frequencies have shifted, and the minimum reflection is now larger:

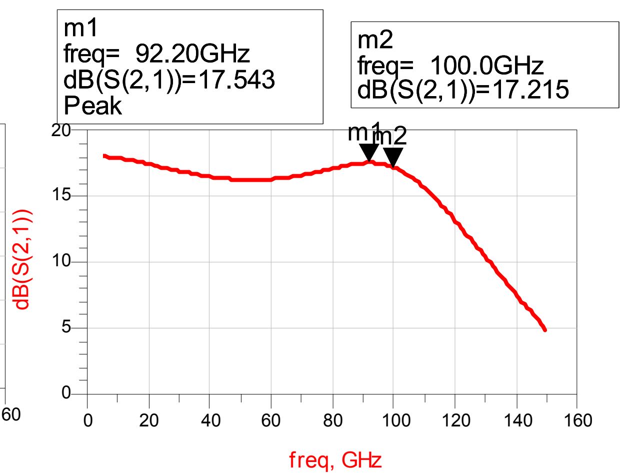




Because of C_{GD}, the input and output couple with each other much more than before, because S_{12} is now non-zero, and there is an extra C_{GD} dependence in S_{21}. Re-matching the transistor require the input and output to be re-tuned in ADS about two times each, resulting in the following circuit and measurements:



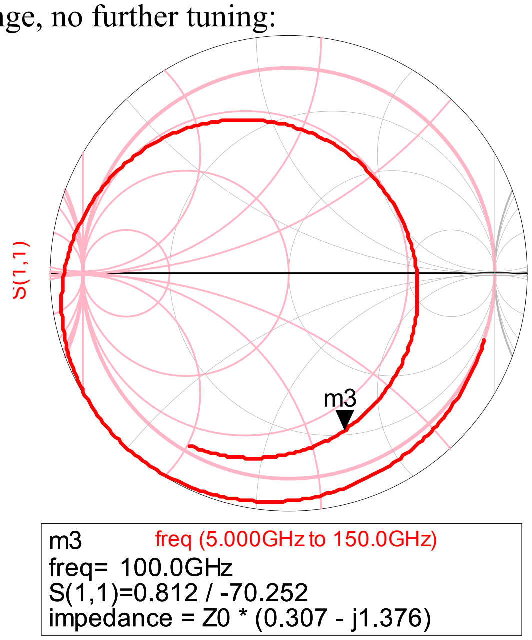


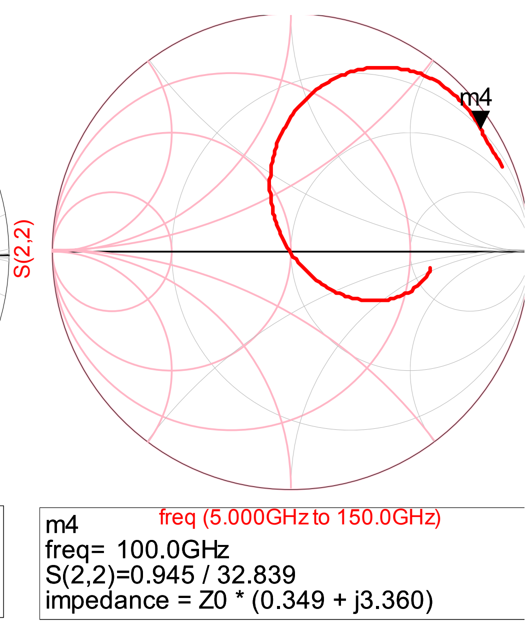


The input and output reflection seem to be as good as before, but the gain was reduced.

E)

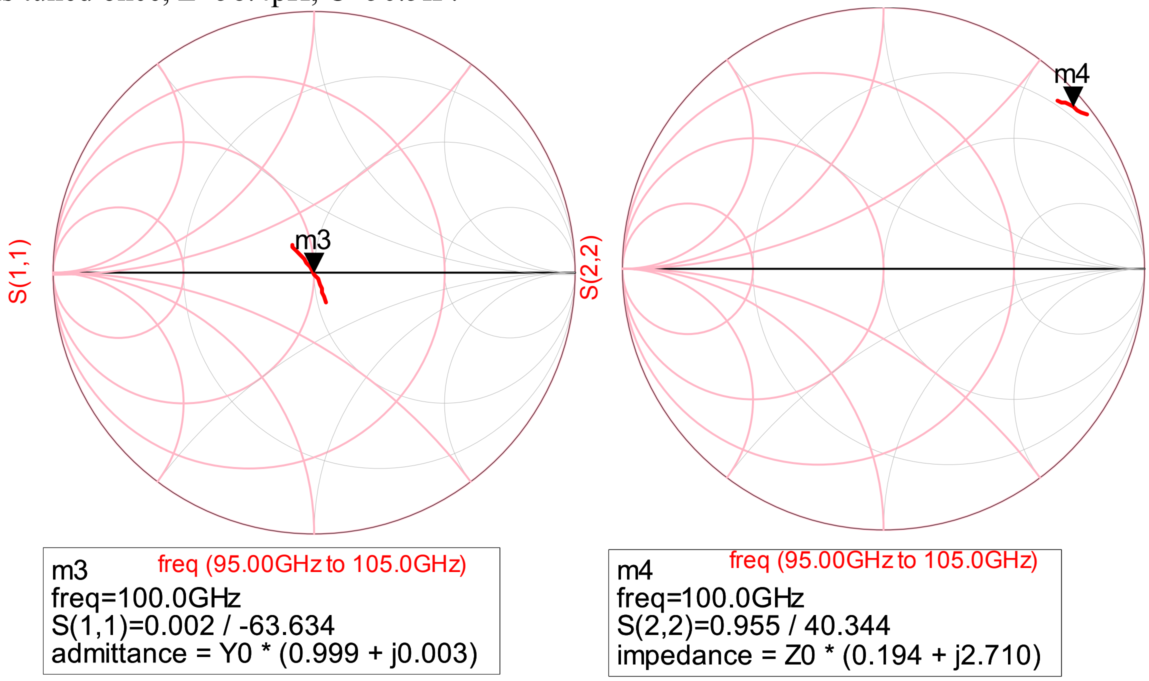
C_{GD} is increased to 5fF and R_I reduced to 5Ω, causing much stronger coupling between the input and output. Results of change, no further tuning:



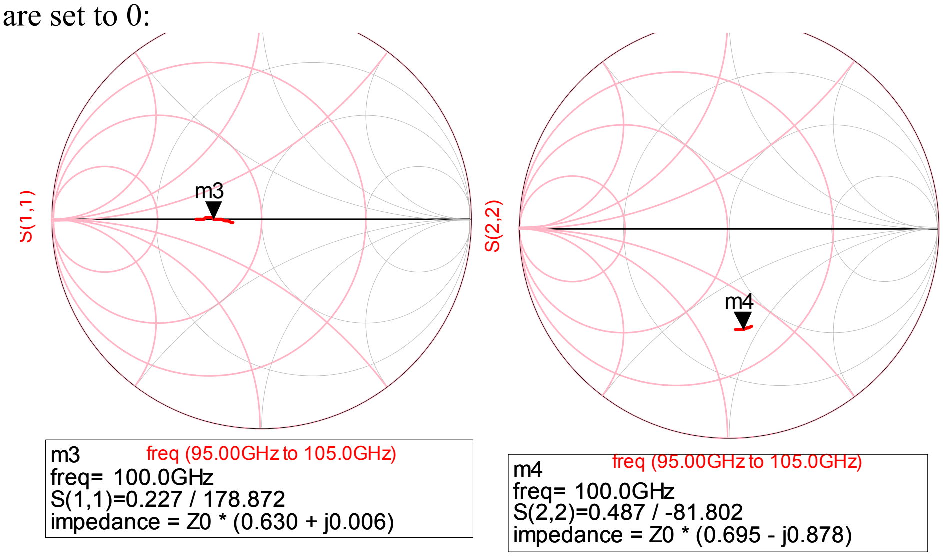


We can see that at certain frequencies the S11 will go outside the unit smith chart, which indicates a negative resistance, which means that an input signal would be reflected at a greater magnitude than before.

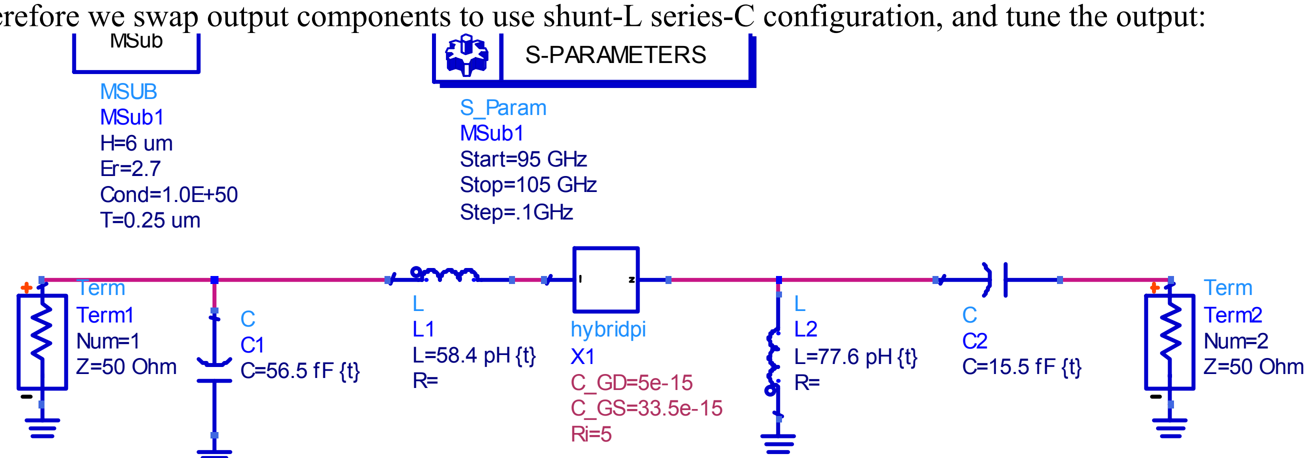
The input is tuned once, L=58.4pH, C=56.5fF:

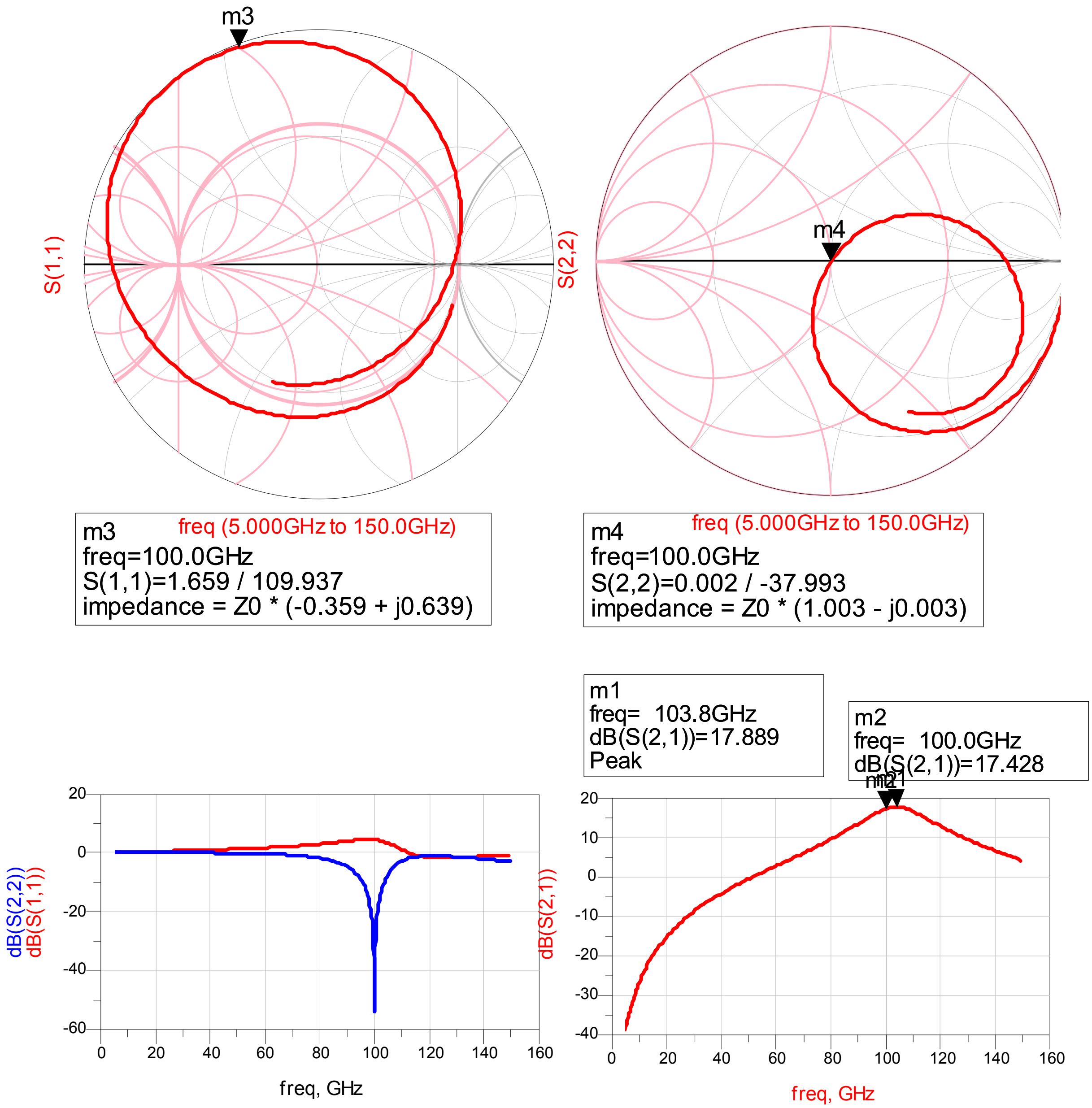


We can see that input is matched, but we can no longer match output with this topology as seen when the values of output L and C are set to 0:



Therefore we swap output components to use shunt-L series-C configuration, and tune the output:





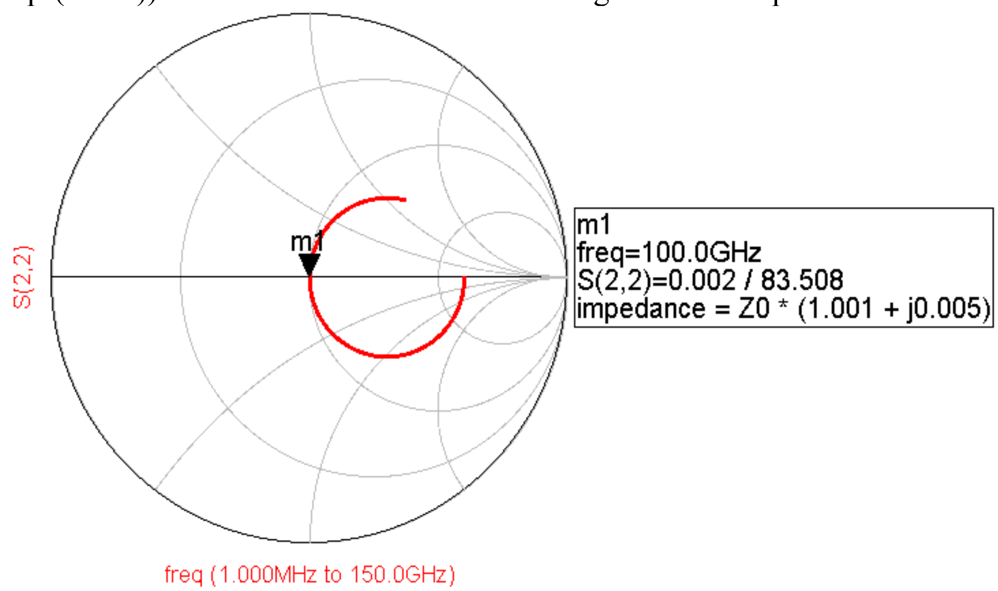
Now that the output is tuned, the input is horribly detuned at the design frequency, and is well outside of the unit smith charg. If we were to continue we would find that it is not possible to match the intput and output simultaneously.

This is because the amplifier is unstable. The Rollet stability factor ("k factor") tells us that an amplifier might be unstable if k<1. This will be covered in more detail during lecture, but what we can see is that such amplifiers cannot be perfectly matched, and we will actually need to select an input and output matching setup which ensures stability by creating a mis-match.

Problem #5

Output: With C_{GD}=0, the output impedance is purely caused by R_{DS} = 200. We design a quarter-wavelength transformer with an impedance $Z_0 = \text{sqrt}(Z_{IN} * Z_L)$, where Z_{IN} is what we are matching to, typically 50Ω, which gives us Z_0 = 100.

LineCalc shows that this is realized on our substrate with a line width of 4.12um, giving an ε_{Eff} = 2.012, thus $L_{\lambda/4}$ = c/(4*100GHz*sqrt(2.021)) = 528um. We see below that we get an almost perfect match.



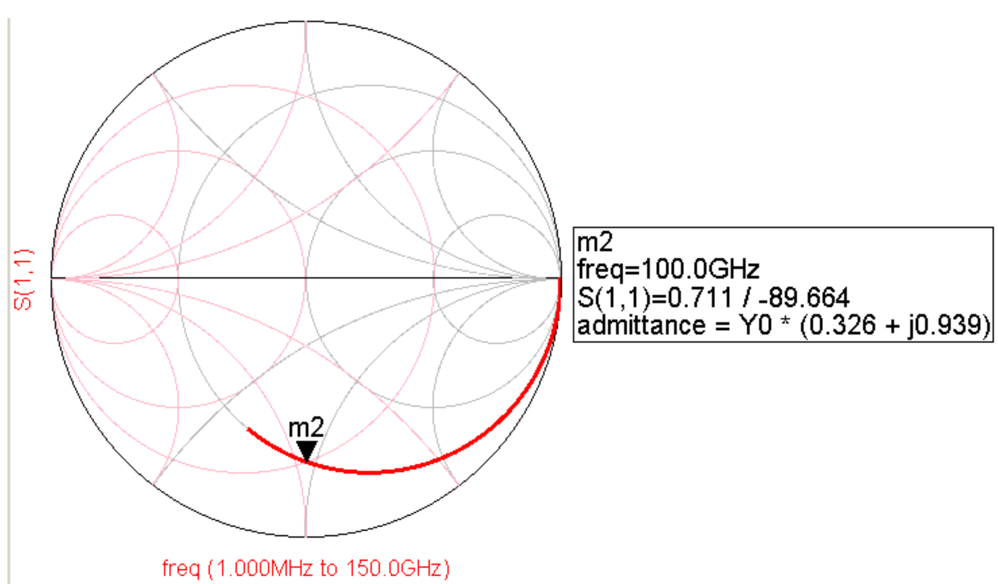
Input: Because the input reflection has a complex component, to properly match it we must first bring the reflection coefficient to the real axis. In this case it is given that we use a high impedance shunt line. This is similar to an inductor to ground, so we will approximately move along a constant conductance circle.

We see in simulation below that at 100GHz we have

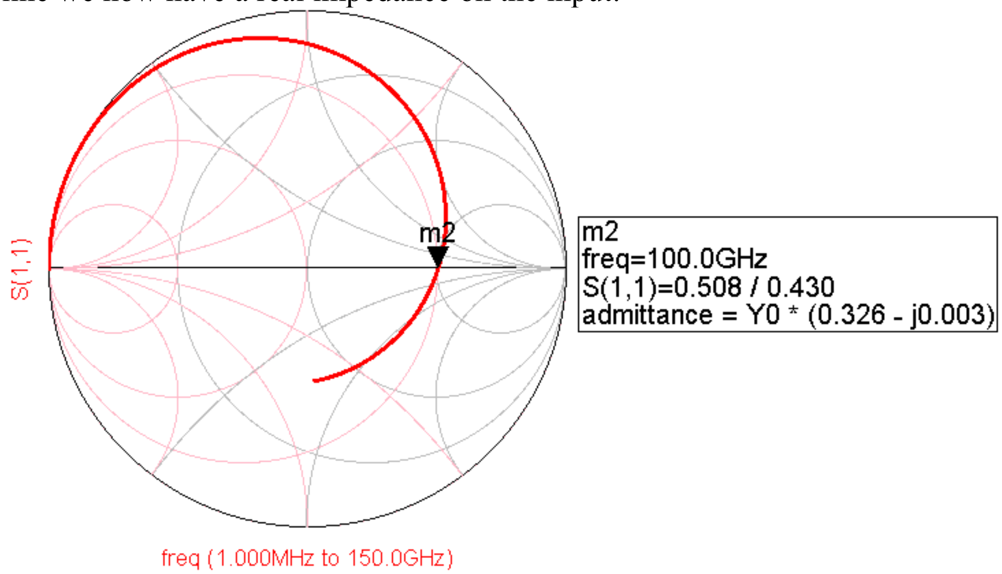
- Y_{input} = (0.326 + j*0.939)/50

To cancel the imaginary part, we can use ADS's tuning feature, or use the fact that the admittance of a shorted line is

- $Y_{SC} + \text{Im}[Y_{IN}] = -j/(Z_0 \tan(\beta d)) + j*.939/50 = 0$
- We are told to use 5um lines, thus Z_0 = 92.3
- d = .0833λ, 29.9deg, with linecalc: 174um



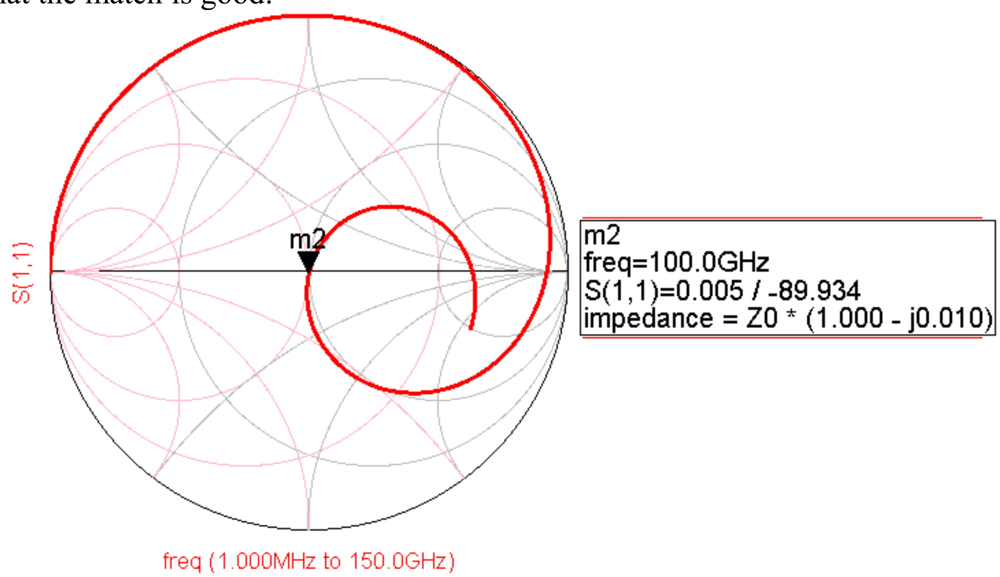
With this series line we now have a real impedance on the input:



To finish the match we use a quarter wave transformer with

- $Z_0 = 50*\text{sqrt}(1/0.326) = 87.5\Omega$
- LineCalc synthesizes with W = 5.655um, L = 522.5um

We see below that the match is good:



Final Circuit: