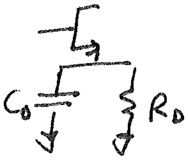
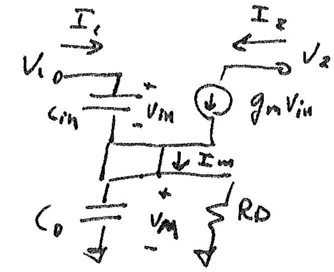
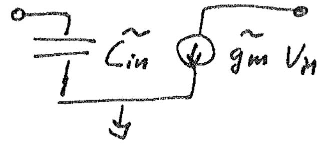
① Show that emitter degeneration  $\Rightarrow$ 

is equivalent to: 

$\tilde{C}_{in} = C_{in}/1+g_m R_D$

$\tilde{g}_m = g_m/1+g_m R_D$

when $C_D R_D = C_{in}/g_m$. Find Y parameters.

- $Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$
- $V_1 = V_M + V_{IN}$
- $I_1 = V_{in}\, j\omega C_{in}$

Given $C_0 R_D = C_{in}/g_m$

$$\left.\begin{aligned} I_M &= V_{IN}(j\omega C_{in} + g_m) \\ V_M &= I_M \cdot \frac{1}{j\omega C_D + 1/R_D} \end{aligned}\right\} V_M = V_{IN} \cdot \frac{j\omega C_{in} + g_m}{j\omega C_D + 1/R_D} = V_{IN} R_D \cdot \frac{j\omega C_{in} + g_m}{j\omega C_{in}/g_m + 1}$$

$$= V_{in} R_D g_m \cdot \frac{j\omega C_{in} + g_m}{j\omega C_{in} + g_m} = V_{in} R_D g_m$$

so, $V_1 = V_{IN} + V_M = V_{IN}(1 + g_m R_D)$

$$Y_{11} = \frac{V_{in}\, j\omega C_{in}}{V_{IN}(1+g_m R_D)} = \boxed{\frac{j\omega C_{in}}{1+g_m R_D} = j\omega \tilde{C}_{in} = Y_{11}}$$

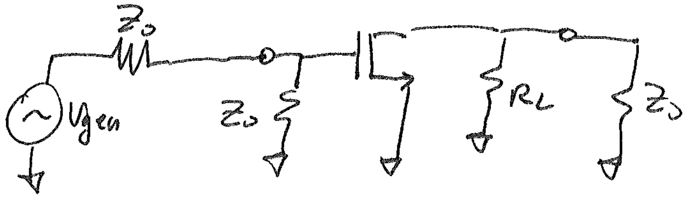
- $Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0} = 0$ since $C_{GD} = 0$
- $Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0} = 0$ since I_2 depends on V_{IN} only
- $Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$

$I_2 = g_m V_{in}$

$V_1 = V_{IN} + V_M = V_{in}(1 + g_m R_D)$

$$\boxed{Y_{21} = \frac{g_m}{1+g_m R_D} = \tilde{g}_m}$$

(2)



$g_m = 1\frac{mS}{\mu m} \cdot W_g \Rightarrow W_g = \frac{g_m}{1\ mS/\mu m}$

$R_i = 0.5/g_m$

$g_m R_{DS} = 20 \Rightarrow R_{DS} = 20/g_m$

$C_{gd} = 0$

$f_T = 200\,GHz \rightarrow f_T \cong g_m/2\pi C_{gs}$

$S_{21} = 10dB$

* Find R_L, W_g, f_{3dB}

- $S_{21} = 20\log|A_v|$
- $A_v = 10^{S_{21}/20} = 3.16$
- From baseband analysis $A_v = g_m R_{L,eq}$ where $R_{L,eq} = R_{DS} \| R_L \| Z_0$
- We must also match $R_{DS}\|R_L$ to $50\,\Omega$ to minimize Reflections

Thus : $50 = \left(\frac{1}{R_{DS}} + \frac{1}{R_L}\right) = \left(\frac{g_m}{20} + 1/R_L\right)$

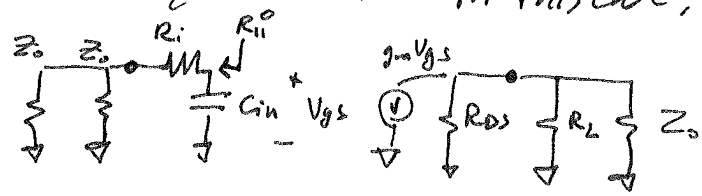
$3.16 = g_m\left(\frac{1}{R_{DS}} + \frac{1}{R_L} + \frac{1}{Z_0}\right) = g_m\left(g_m/20 + 1/R_L + 1/50\right)$

Solve by hand or computer:

$\boxed{g_m = 0.1265 \Rightarrow W_g = 126.5\,\mu m \quad R_L = 73.12\,\Omega}$

- f_{3dB} is determined by dominant, or in this case, the only pole: a_1

Equiv. Ckt:



$R_i = 0.5/g_m = 3.9$

$C_{in} = g_m/2\pi f_T = 100\,fF$

Using MOTC with respect to C_{in}: $R_{11}^{o} = R_i + Z_o/2 = 28.9$

$a_1 = R_{11}^{o} C_{in} = 2.89\,ps$

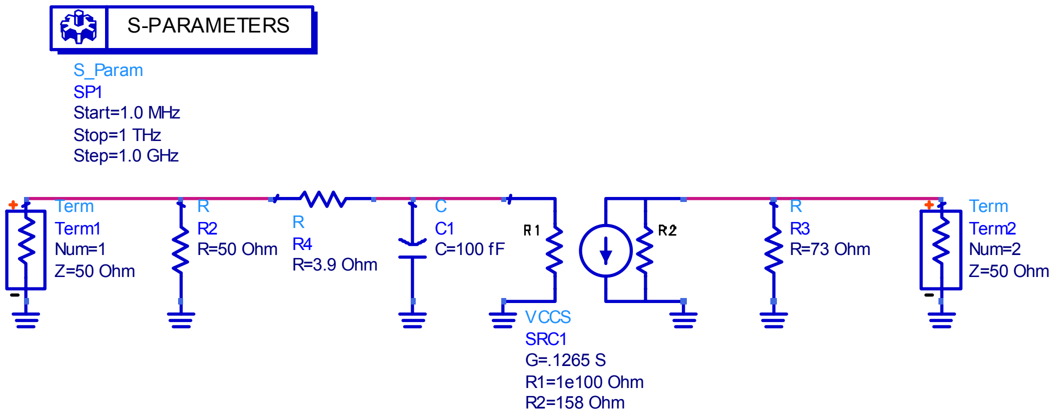
$\boxed{f_{3dB} = \frac{1}{2\pi(2.89ps)} = 55.1\,GHz}$

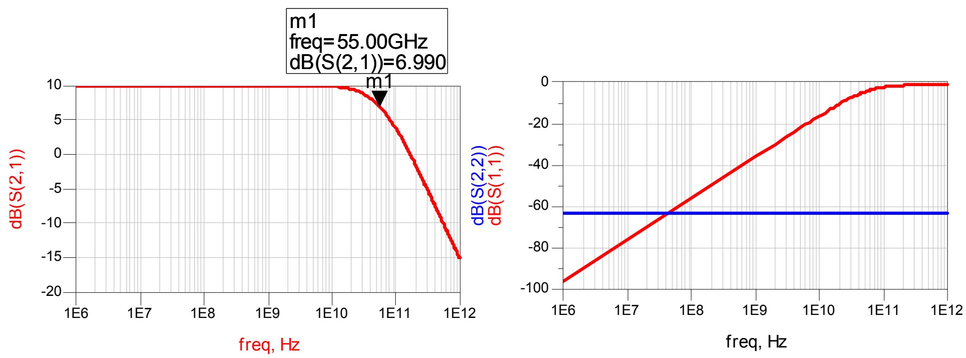
For this simple case with 1 capacitor the pole is exactly where we expect : $f_p = 1/2\pi RC$

MOTC will be helpful in more complex circuits.

<u>Problem #2</u>

Derived parameters entered in schematic:



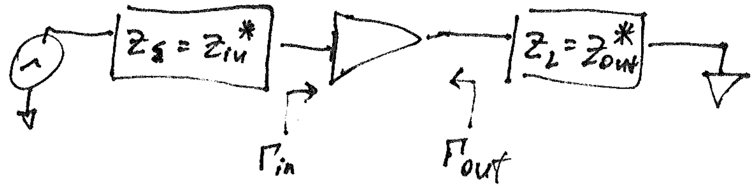


Results: The spec of 10dB low frequency gain is met, with f_{3dB} at 55GHz as predicted by analysis.

Input and output are well matched at low frequencies by the resistive termination. However, S_{11} increases at 20dB/dec because of the input capacitor.

(3) For Problem 2's transistor find
Max available power gain (MAG) @ 50 GHz
and f_{max} (where MAG = 1)

For MAG, input & ouput are both conjugate matched,
so Γ_{in} and $\Gamma_{out} = 0$

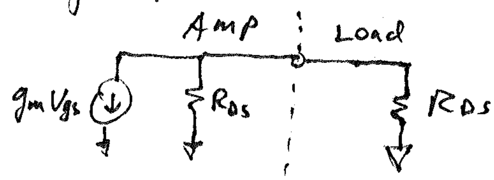


$C_{gs} = 100\,fF$
$C_{gd} = 0$
$R_{in} = 3.9\,\Omega$
$R_{DS} = 158\,\Omega$
$g_m = 0.1265$

$P_{in} = I_{in}^2 R_{in}$, $V_{gs} = I_{in}\,\omega C_{gs}$

$P_{out} = I_{out}^2 R_{out}$ For matching output we need $Z_L = R_{DS}$

so $I_{out} = g_m V_{gs}/2$



$P_{out} = \left(\frac{g_m V_{gs}}{2}\right)^2 R_{DS}$

$$\underbrace{MAG = \frac{P_{out}}{P_{in}} = \frac{g_m^2 R_{DS}}{4\omega^2 C_{gs}^2 R_{in}}}$$

$$MAG(50\,GHz) = \frac{(0.1265)^2\,158}{4\times(2\pi\times 50E9)^2(100E{-}15)^2\,3.9} = 164.2$$

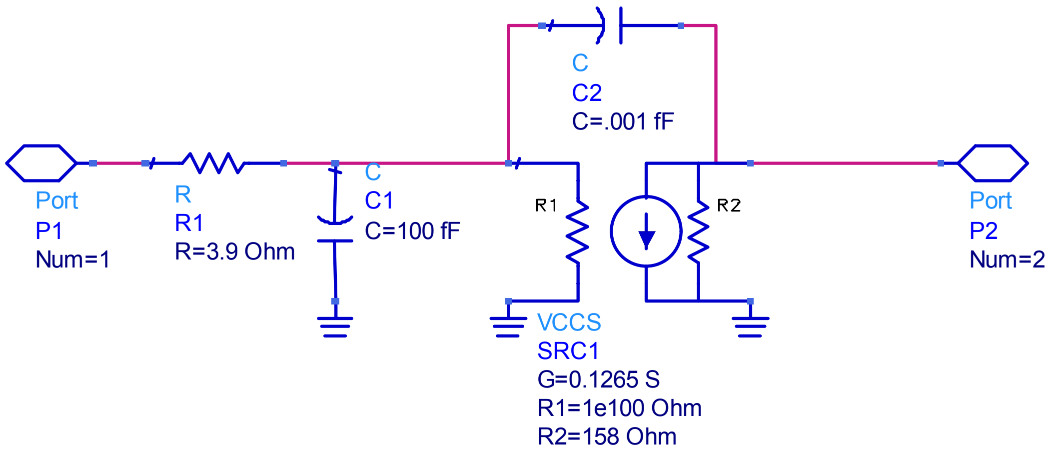
$$10\cdot\log 164.2 = \boxed{20\,dB = MAG(50\,GHz)}$$

→ ADS gives 22 dB

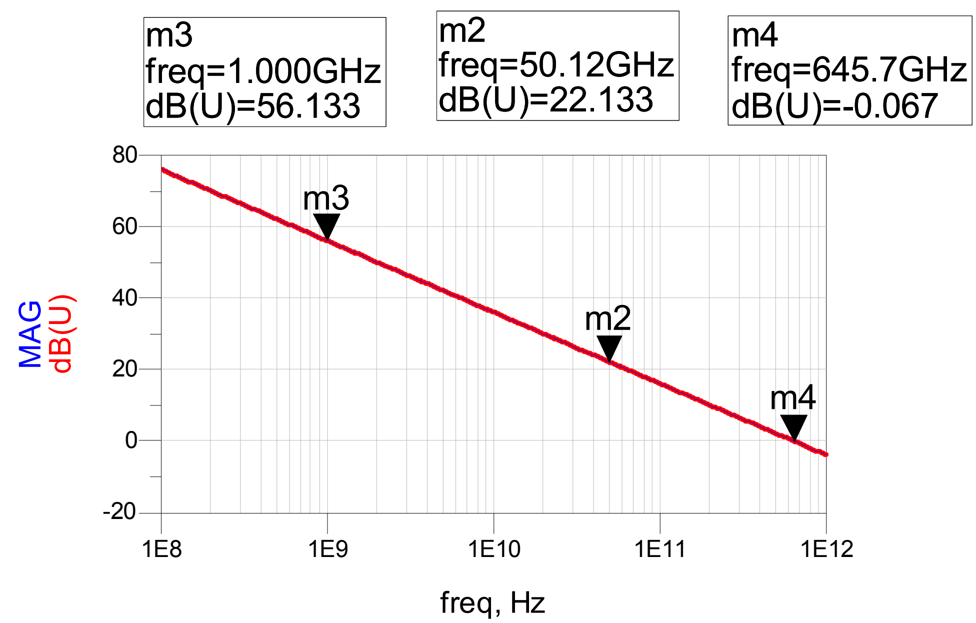
$$@\ f_{max}\quad MAG \equiv 1 = \frac{g_m^2 R_{DS}}{4\omega_{max}^2 C_{gs} R_{in}}$$

$$\Rightarrow \underbrace{f_{max} = \frac{g_m}{4\pi C_{gs}}\sqrt{\frac{R_{DS}}{R_{in}}}} = \boxed{641\,GHz = f_{max}}$$

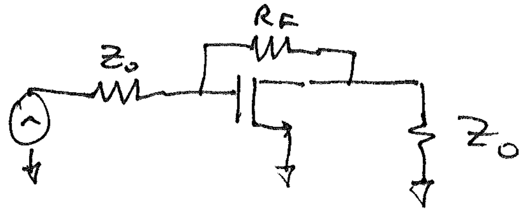
→ ADS gives ~645 GHz



Tested with the Gain-testbench file which Prof. Rodwell provides on course website.



(4)



Feedback amp w/ same FET parameters as problem 2.
Let $R_i = R_{DS} = 0$ to simplify.

Given: $S_{21} = 10dB$
$\Rightarrow A_v = 3.16$

To match to 50Ω so $S_{11} = 0$ @ low freq.:

$\rightarrow R_F = Z_o(1 + A_v) = 208\,\Omega$

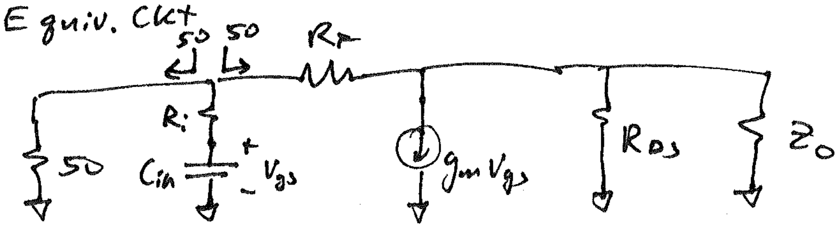
$\rightarrow g_m = (1 + A_v)/Z_o = 0.083\,S$

$\rightarrow W_g = \dfrac{g_m}{1\,mS/\mu m} = 83\,\mu m$

$\rightarrow C_{in} = \dfrac{g_m}{2\pi f_T} = 66\,fF$

<u>3dB BW</u>

Equiv. ckt



$R_i = \dfrac{0.5}{g_m} = 6\,\Omega$

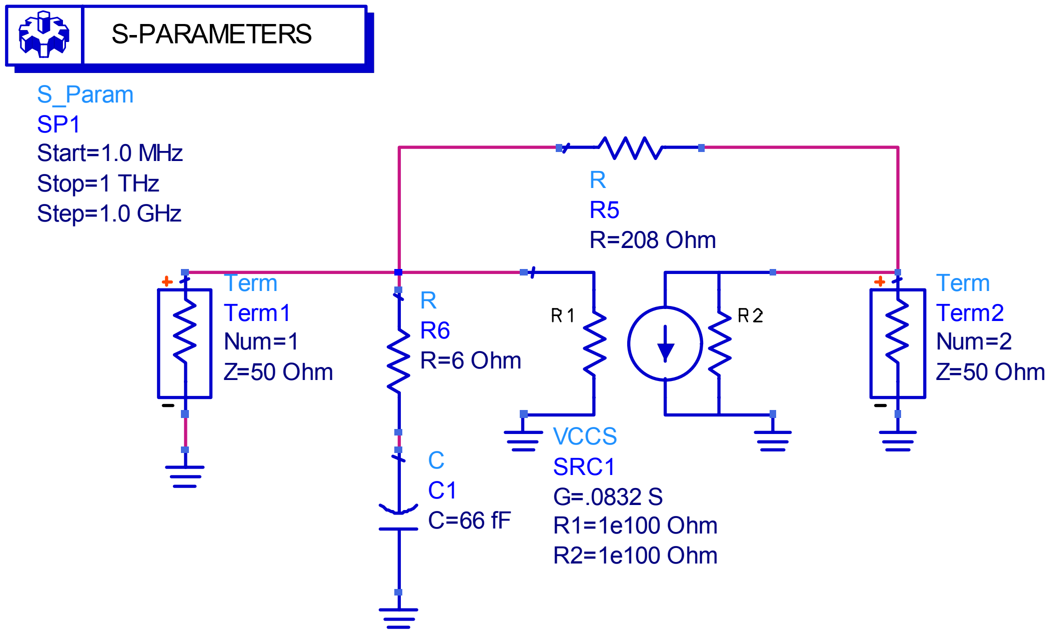
By definition of our analysis, if we neglect R_i
then R_{11}^0 for $C_{in} = 50 // 50 = 25$

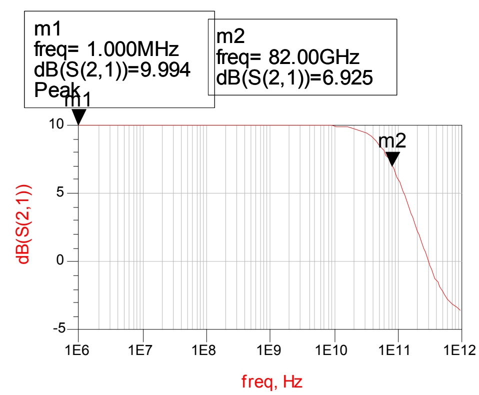
so $f_{3dB} = \dfrac{1}{2\pi(25)(66fF)} = 96.5\,GHz$

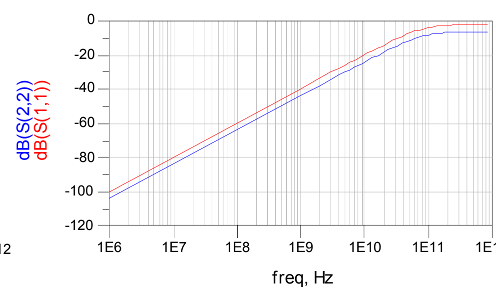
this is a little high because R_i was neglected,
so add it back in: $R_{11}^0 = 6 + 50//50 = 31$

$$\underbrace{f_{3dB} = \frac{1}{2\pi(31)(66fF)} = 77.5\,GHz}_{\text{A much better match with simulation}}$$

<u>Problem #4</u>







We observe that the low frequency gain is 10dB as we designed it for. The match is quite good at low frequencies, the input and output reflections both increase at 20dB/dec.

The f_{3dB} is about 82 GHz, a decent match with our calculations.