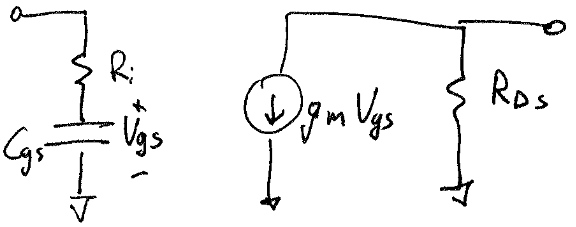
1



$g_m = 1 \frac{mS}{\mu m} \cdot W_g$

$R_i = 0.5/g_m$

$C_{gd} = 0$

$f_T = 200$ GHz

$f_{max} = 400$ GHz

Use parameters found in (2):

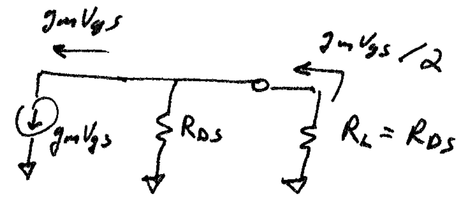
$g_m = 0.209$

$R_i = 2.39$

$R_{DS} = 38.27$

$C_{gs} = 166 fF$

Find MAG



- $P_{out} = I_{out}^2 R_L$

$$= \frac{1}{4} g_m^2 V_{gs}^2 R_{DS}$$

$$= \frac{I_{in}^2 g_m^2 R_{DS}}{4 \omega^2 C_{gs}^2}$$

For MAG output is matched

& $I_{out} = g_m V_{gs}/2$

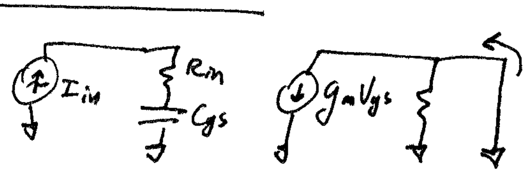
& $V_{gs} = I_{in} \frac{1}{j\omega C_{gs}}$

- $P_{in} = I_{in}^2 R_i$

- $MAG = \frac{P_{out}}{P_{in}} = \frac{g_m^2 R_{DS}}{4 \omega^2 R_i C_{gs}^2}$

Find Short-Circuit Current Gain

Short output:



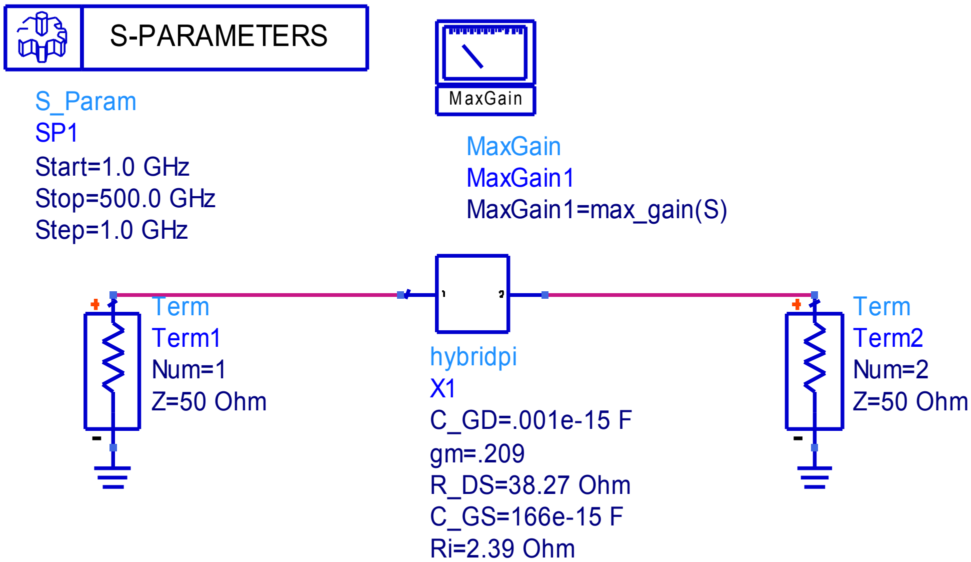
$I_{out} = g_m V_{gs} = g_m I_{in} \cdot \frac{1}{j\omega C_{gs}}$

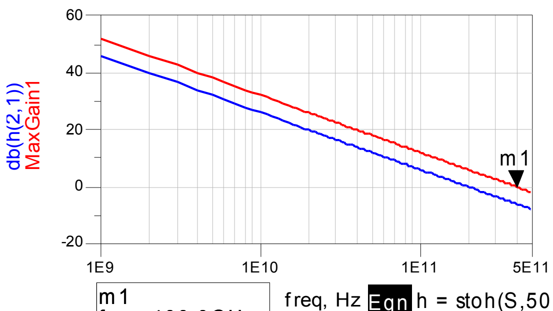
$$\frac{I_{out}}{I_{in}} = \frac{g_m}{j\omega C_{gs}}$$

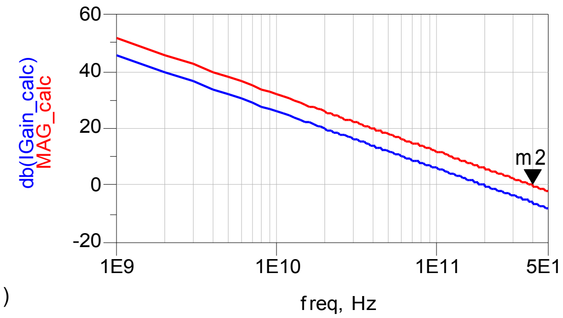
Note: $h_{21} \equiv \left.\frac{I_2}{I_1}\right|_{V_2=0}$ = Short Circuit current gain

When simulating use stoh() function to find current gain

Problem #1







m1
freq=400.0GHz
MaxGain1=0.020

m2
freq=400.0GHz
MAG_calc=0.020

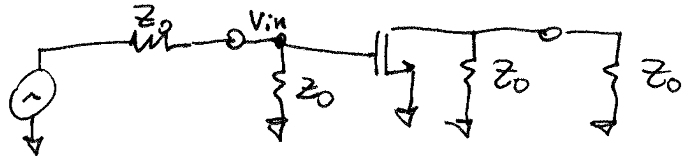
Eqn h = stoh(S,50)

Eqn IGain_calc = .209/(j*2*pi*freq*166e-15)

Eqn MAG_calc = 10*log(.209**2*38.27/(4*2.39*(2*pi*freq*166e-15)**2))

Simulated MAG and current gain are on left, calculated value are on the right. The agreement between the two is extremely good.

(2) For a resistively terminated amplifier find W_g which gives 10dB low frequency gain and determine f_{3dB}. Compare with simulation.



$g_m = \frac{1\,mS}{\mu m} W_g$
$R_i = 0.5/g_m$
$G_d = 0$
$f_T = 200\,GHz$
$f_{max} = 400\,GHz$

• $S_{21} = 10dB = 2\frac{V_{out}}{V_{gen}} = 3.16$

• $\frac{V_{out}}{V_{gen}} = \frac{1}{2} g_m R_L$, since $V_{in} = \frac{1}{2} V_{gen}$

$\frac{2V_{out}}{V_{gen}} = g_m R_L = 3.16 = S_{21}$ $\quad R_L = R_{DS} \| Z_0/2$

Recall from HW4 that @ f_{max} $MAG = 1 = \frac{g_m^2 R_{DS}}{16\pi^2 f_{max}^2 C_{gs}^2 R_{in}}$

$\Rightarrow R_{DS} = 16\left(\frac{\pi f_{max} C_{gs}}{g_m}\right)^2 \cdot R_{in}$ $\quad C_{gs} = \frac{g_m}{2\pi f_T}$

$= 16\left(\frac{\pi f_{max}\, g_m/2\pi f_T}{g_m}\right)^2$

$= 4\left(\frac{f_{max}}{f_T}\right)^2 R_{in}$ $\quad R_{in} = 0.5/g_m$

$= 4(2)^2\, 0.5/g_m$

$R_{DS} = 8/g_m$

So, $S_{21} = 3.16 = g_m\left(\frac{8/g_m \times 25}{8/g_m + 25}\right) = \frac{200}{\frac{8}{g_m} + 25}$

$\Rightarrow g_m = 0.209$

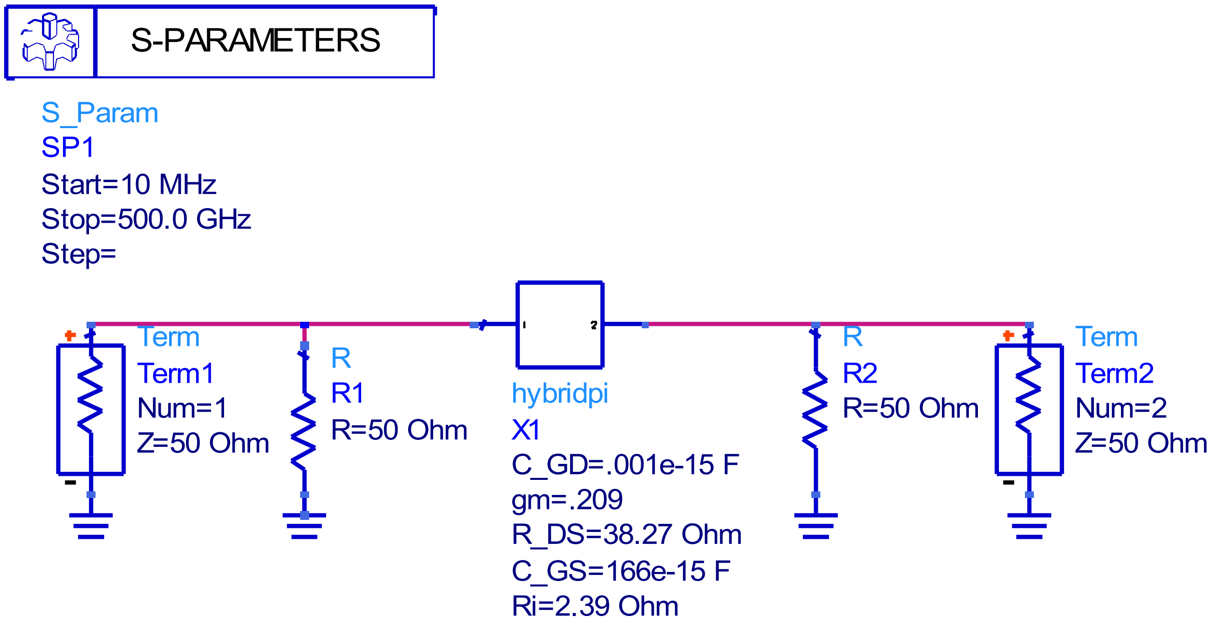
$\Rightarrow W_g = 209\,\mu m$

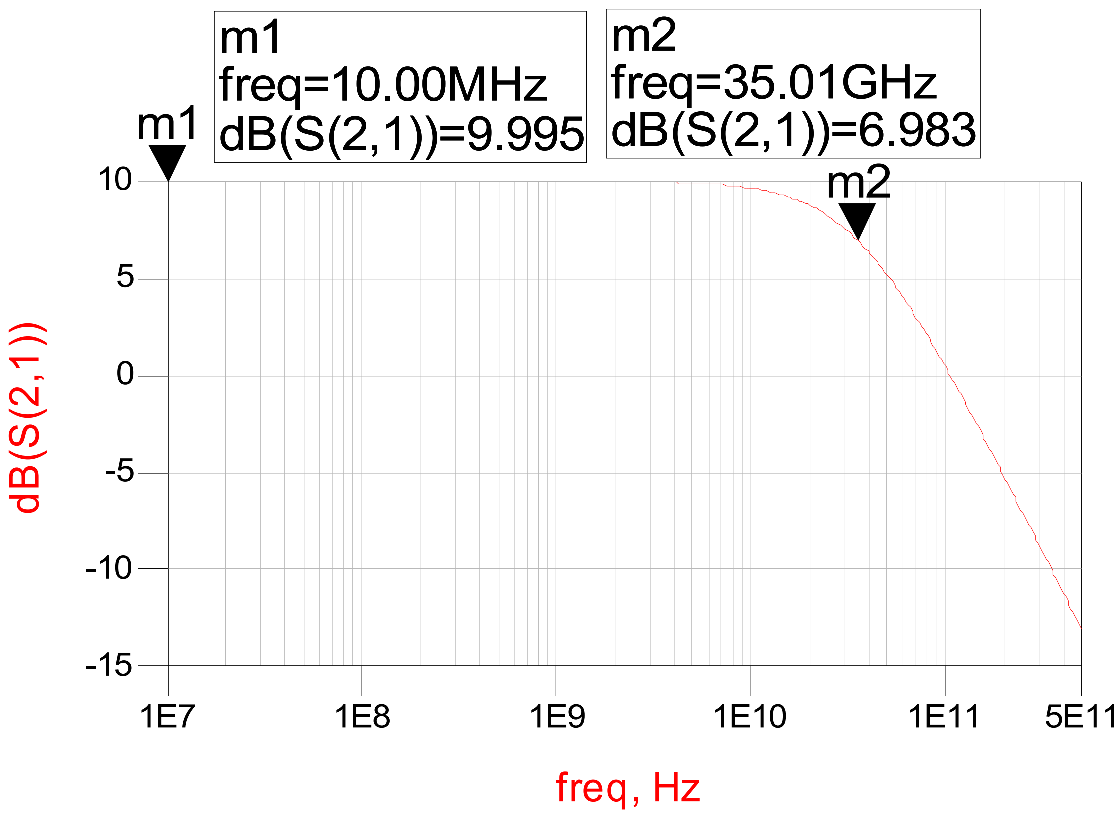
So, $C_{gs} = 166\,fF$, $R_i = 0.5/g_m = 2.39$, $R_{DS} = 8/g_m = 38.27$

$f_{3dB} = \frac{1}{2\pi(R_i + Z_0/2)C_{gs}} = \frac{1}{2\pi(2.39+25)\cdot 166\,fF} = 35\,GHz$

(agrees w/ simulation)

Problem #2



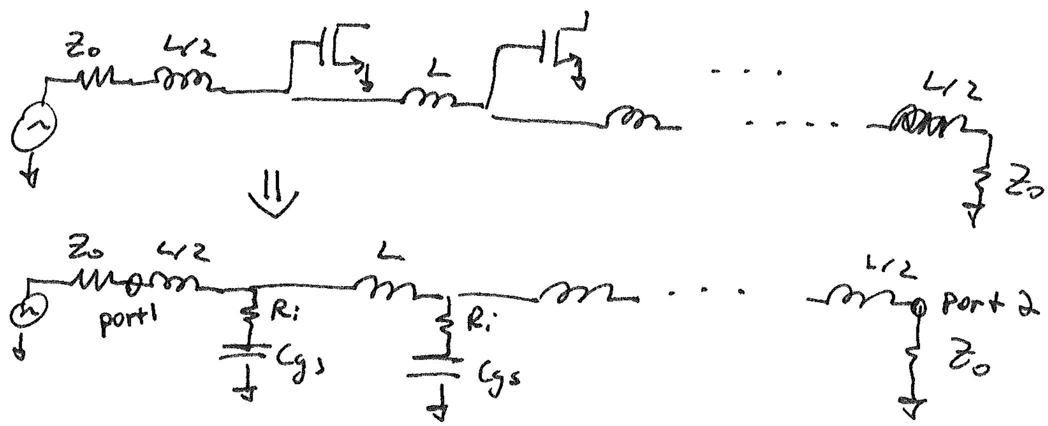


The low frequency gain of 10dB has been achieved. The calculated 3dB frequency of 35 GHz agrees quite well with the simulation.

(3) Take (2)'s device and split into N parallel devices with $W_{g,new} = W_{g,old}/4$

Let $N=4$.

For input, find f_{Bragg}, L, and freq-dependent loss



With new $W_g = 52.25$

$\Rightarrow R_i = 9.56\,\Omega,\ C_{gs} = 41.5\,fF$

We must have $Z_0 = \sqrt{\frac{L}{C}} = 50\,\Omega$

$\Rightarrow L = 50^2 \cdot C_{gs}$

$\boxed{L = 104\,pH}$

$\boxed{f_{Bragg} = \frac{1}{\pi\sqrt{LC}} = 153\ GHz}$

Loss:

Loss per section $A_g \cong \omega^2 R_i C_{gs}^2 \frac{Z_0}{2}$

$= 2R_i Z_0 (\pi f C_{gs})^2$

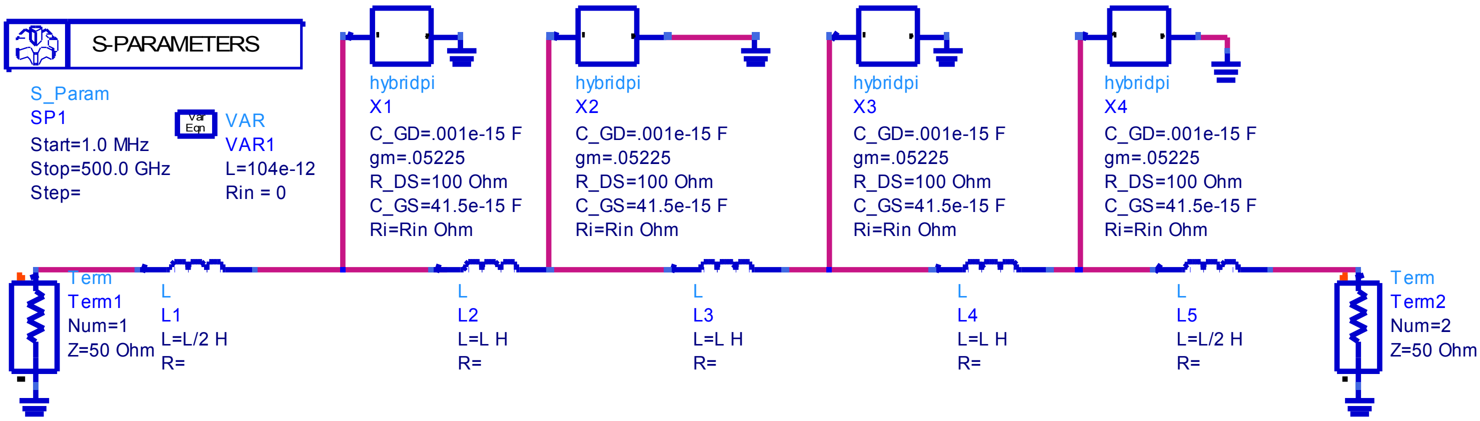
Loss from port 1 to 2 $= e^{-N \cdot A_g}$

$\boxed{S21\,(dB) = 20\log\left[\exp(-4 \cdot 2 \cdot R_i \cdot Z_0 (\pi f C_{gs})^2)\right]}$

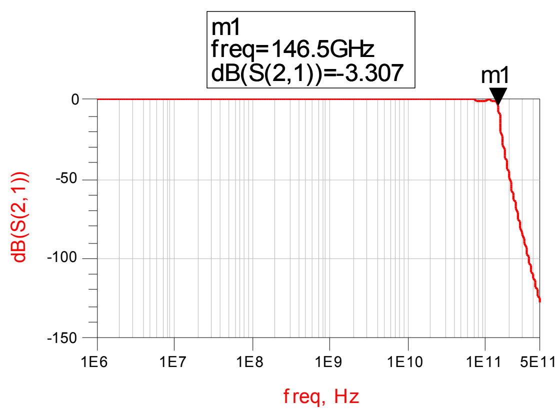
↑ Plot in ADS

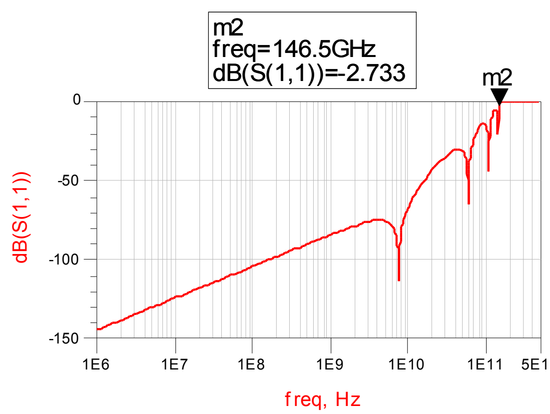
Problem #3

First simulate with Rin = 0 to verify cutoff frequency:

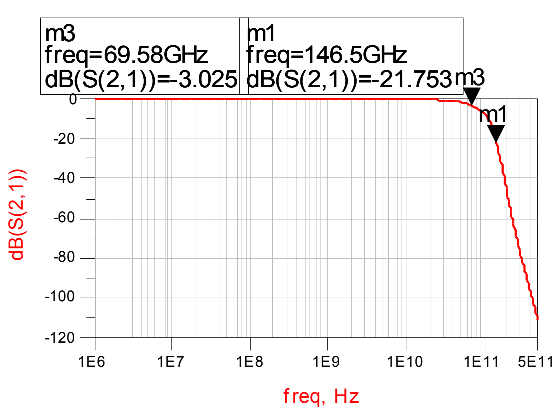


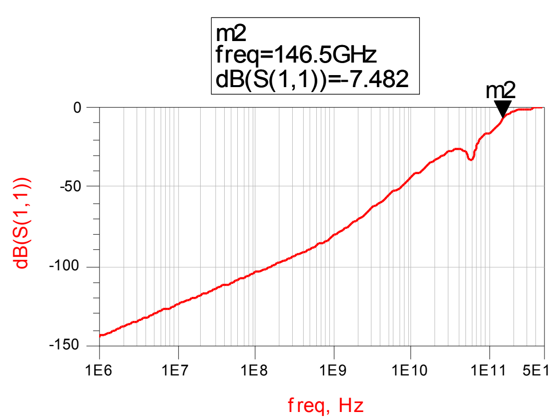
We can see that the calculated Bragg frequency is quite close to the simulation. The forward gain is also very close to unity (no attenuation) up until the cutoff frequency after which it decreases incredibly quickly. The input reflection is also quite small at low frequencies, but it increases steadily, approaching unity at the cutoff frequency.





Simulation is performed again with Rin = 9.56 Ohms as calculated. As we might expect, it lowers the 3dB bandwidth, in this case to about 70GHz. It also damps out some of the spikes seen in the input reflection.

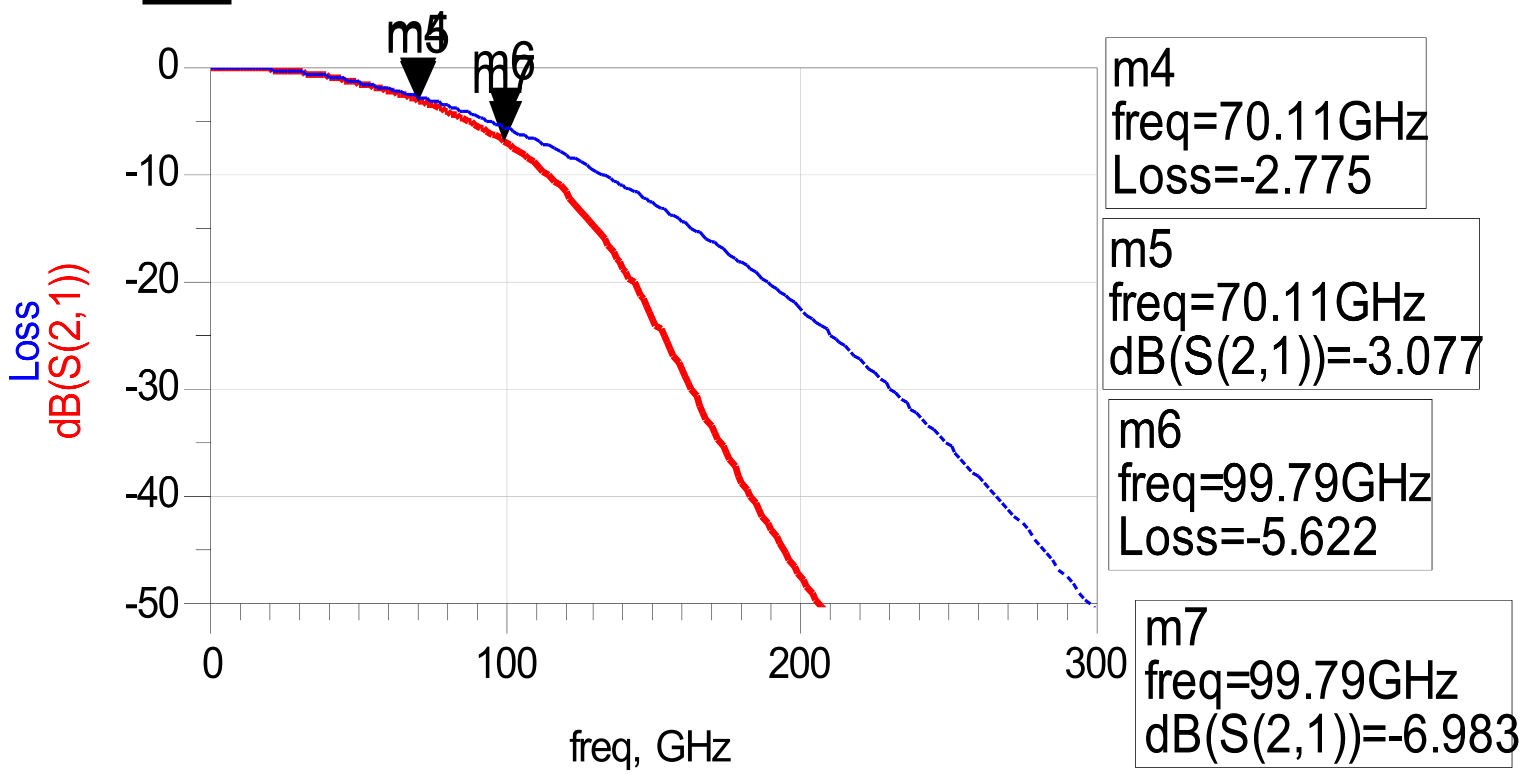




The calculated loss is plotted with S21. At low frequencies the agreement is good between the two, but as frequency increases the agreement worsens. This is a result of multiple approximations which were taken in the derivation of the loss equation.

However it can be seen that this approximate loss equation gives a good prediction of the 3dB bandwidth of the input line, of 74GHz, versus the simulated 70GHz. Since the loss after 3dB is generally not very important, it can be a very useful design equation.

Eqn Loss = 20*log10(exp(-4*(2*9.56*50*(pi*41.5e-15*freq)**2)))



(4) Found in (3) that f_{3dB} due to loss is 20GHz

We want to find N s.t. $f_{Bragg} = f_{3dB}$.

~~C_{gs}=~~ $f_{Bragg} = \frac{1}{\pi\sqrt{L C_{gs}}} = f_{3dB}$

$Z_0 = \sqrt{\frac{L}{C_{gs}}} \Rightarrow L = C_{gs} \cdot 50^2$

Combine & get: $C_{gs} = \frac{1}{\pi Z_0 f_{3dB}} = 91fF$

$L = 227pH$

From C_{gs} we find other FET parameters

$g_m = C_{gs}\, 2\pi f_T = 0.114\ S$

$R_i = 0.5/g_m = 4.37\,\Omega$

$R_{DS} = 8/g_m = 70\,\Omega$

Total loss @ $f_{3dB} = \exp(-N\, 2R_i Z_0 (\pi f_{3dB} C_{gs})^2)$

Recall $\log_{10}(x) = \frac{\ln(x)}{\ln(10)}$

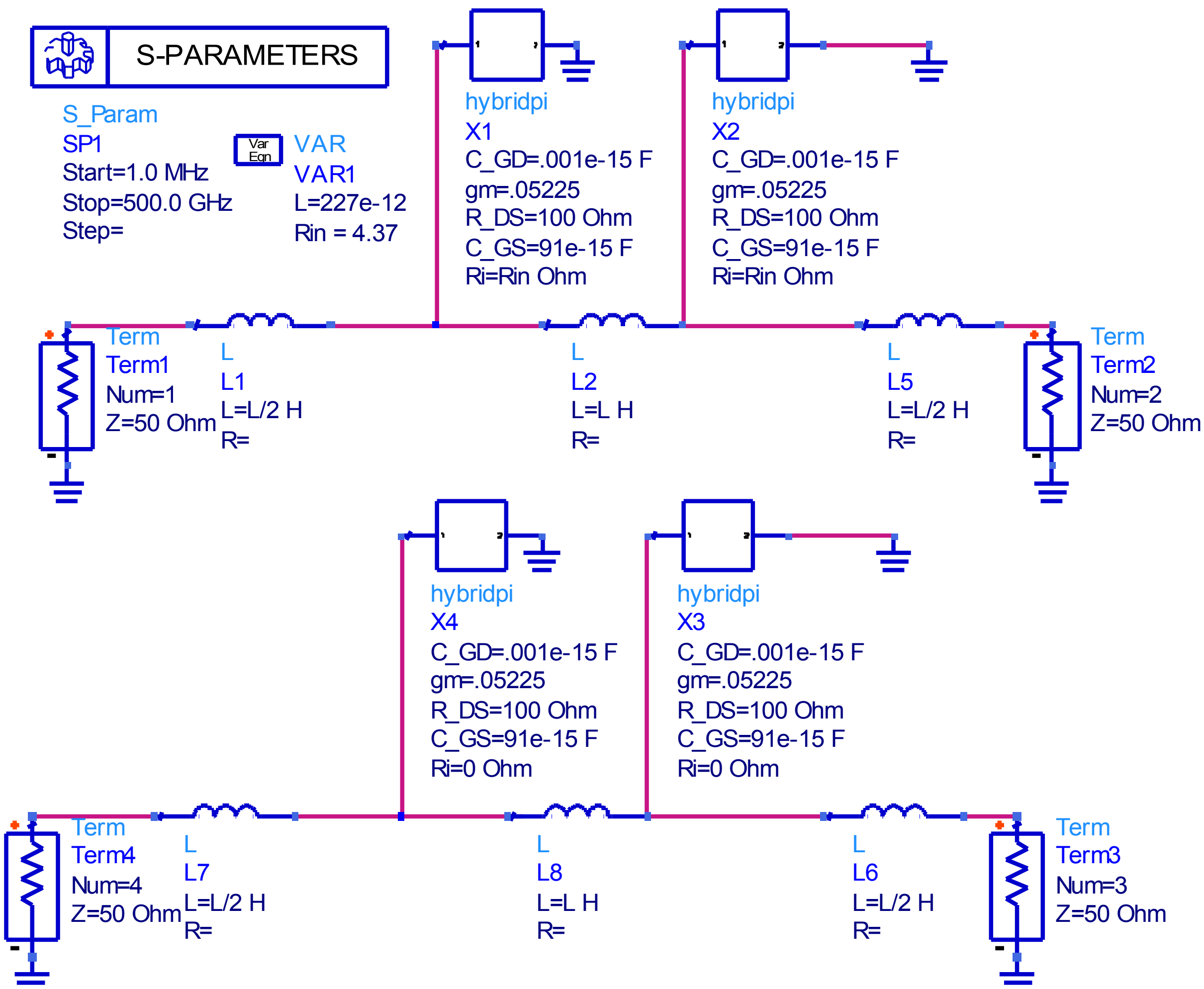
$Loss = -3dB = \frac{20}{\ln(10)} \cdot N\, 2R_i Z_0 (\pi f_{3dB} C_{gs})^2$

$N = \frac{3 \ln 10}{40 R_i Z_0 (\pi f_{3dB} C_{gs})^2}$

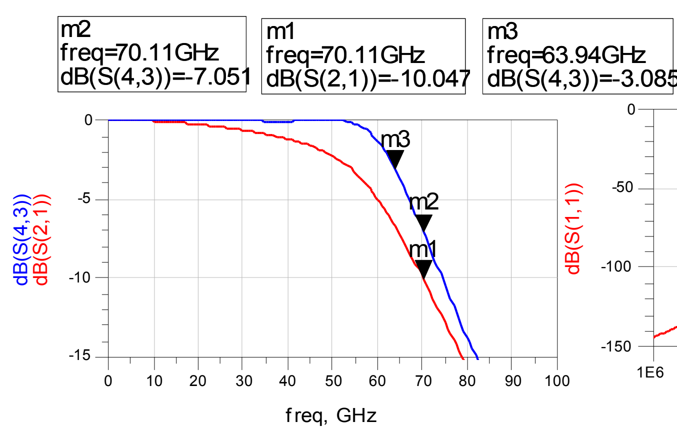
$N = 1.97 \simeq \boxed{2}$

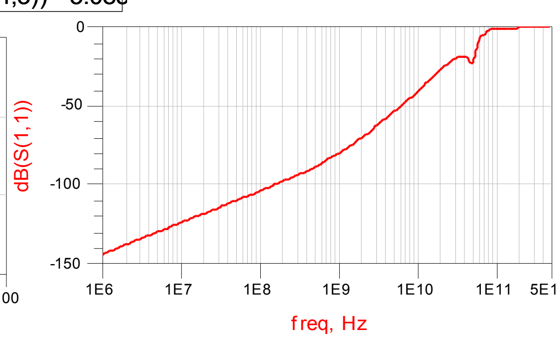
Problem #4

Two circuits are tested: on the top with ports 1 and 2 is a circuit including the Ri loss. On the bottom with ports 4 and 3 is a circuit with Ri=0.



We can see below that for the lossless case (S43) the bragg frequency is 6GHz lower than predicted, at 64GHz. But at 70GHz we can see that the loss does indeed lower S21 3dB below S43, so the bragg cutoff and loss 3dB bandwidth have been placed very close to each other.





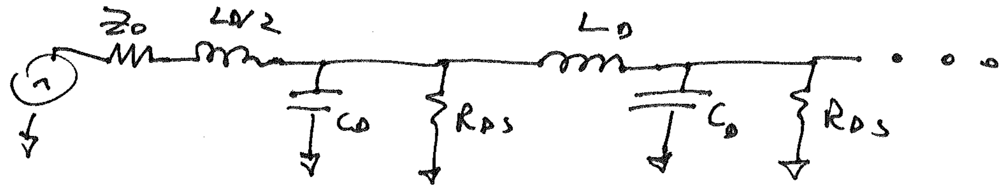
(5) Find L & C to obtain same per-section delay on output line.

Easy, just use same values: $L = 227pH$

$C = 91fF$

$$f_{Bragg} = \frac{1}{\pi\sqrt{LC}} = 70GHz$$

Find attenuation on output line:



← This causes loss.

It is no longer frequency-dependent

$$A_d \cong \frac{Z_0}{2R_{DS}} = 0.357 \quad \text{(per section, given in notes)}$$

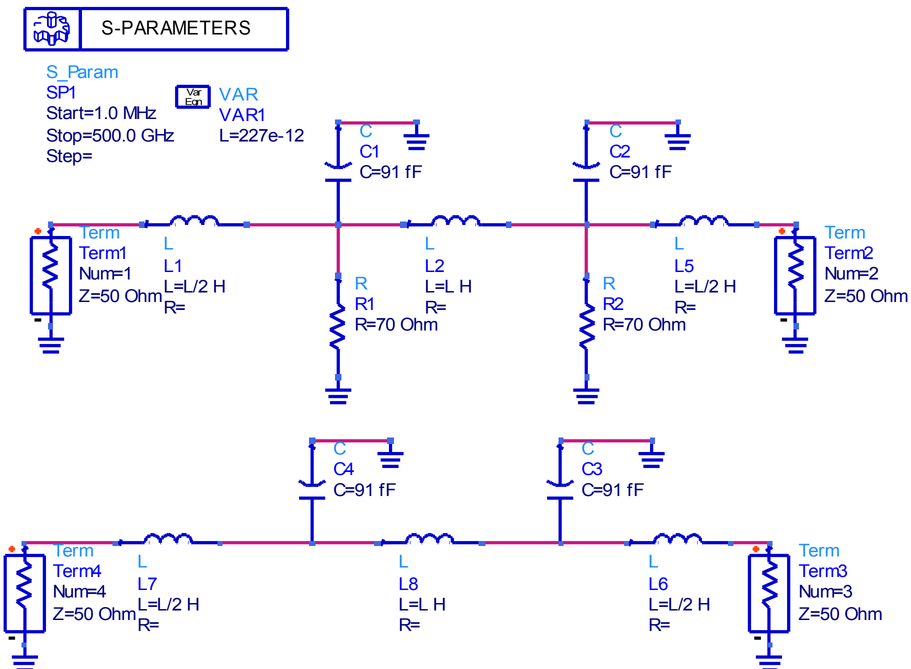
$$S_{21} = 20\log e^{-2A_d} = \frac{20}{\ln 10}(-2A_D) = \underline{-6.2dB}$$

↑ N=2 from (4)

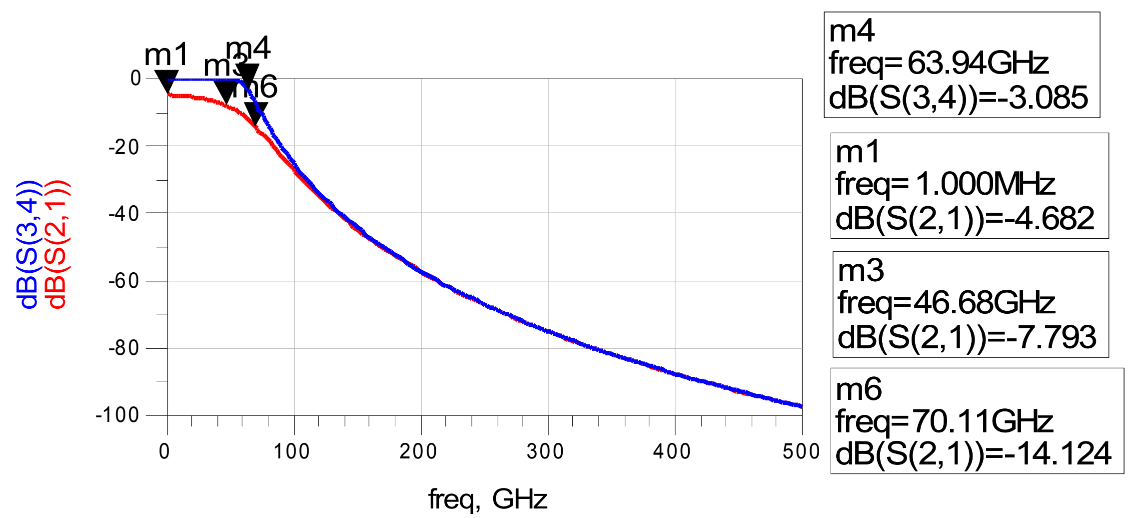
The loss is significant because R_{DS} is small.

Problem #5

Taking same L and C values from problem 4 is a simple way to ensure delays are matched.

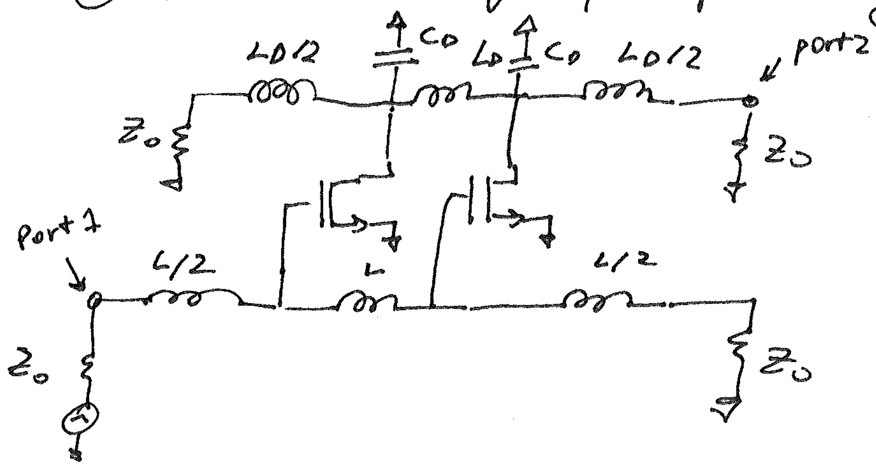


The unloaded transmission line has a cutoff frequency of 64GHz, as it did in problem 4. We can see that the low frequency loss of 4.7dB is 1.5dB less than what was calculated. The approximation that was used to find the drain loss is better as RDS is greater than Z0. We also see the cutoff frequency lower than expected at 47GHz. This is due of the large effect that that a low RDS has on the artificial transmission line.



Comparing plots from the last problem, the loss on the output transmission line is higher than the input, according to the notes it is usually the reverse, but that assumes a high drain-source resistance.

⑥ Find the frequency-dependent gain.



FET#1

Gate: $V_g e^{-A_g/2}$

Drain: $g_m V_g e^{-A_g/2}$

followed to port 2: ~~$g_m V_g A_e$~~ $g_m V_g e^{-A_g \cdot \frac{1}{2}} e^{-A_d \cdot \frac{3}{2}}$

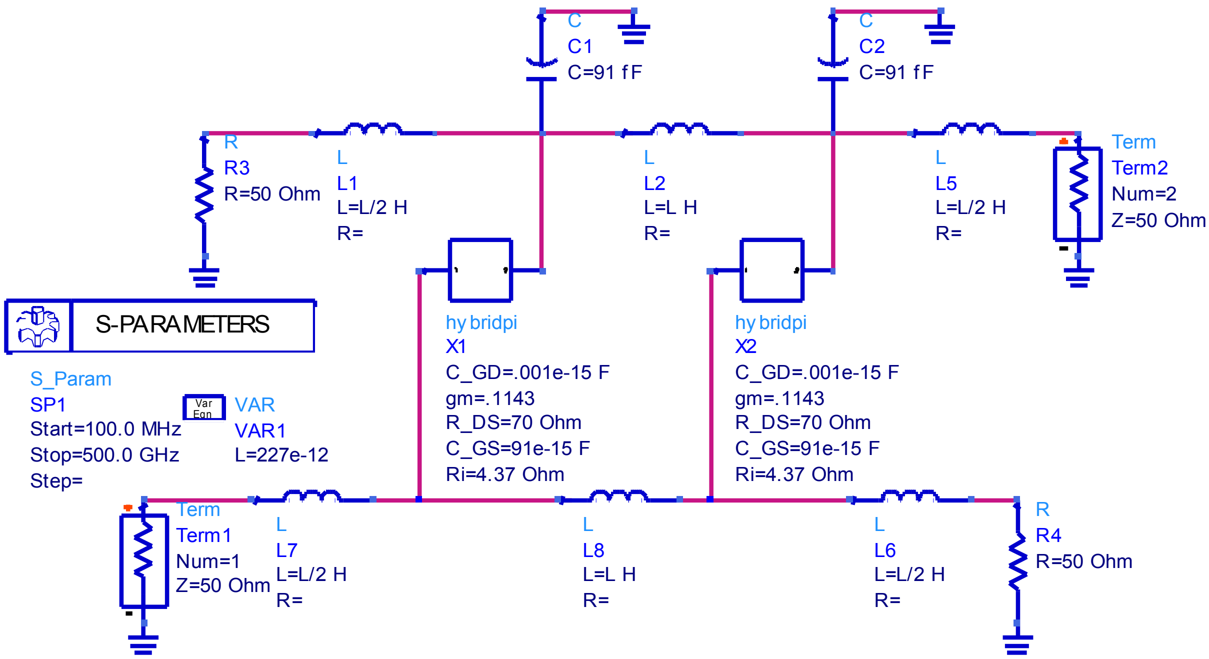
FET 2

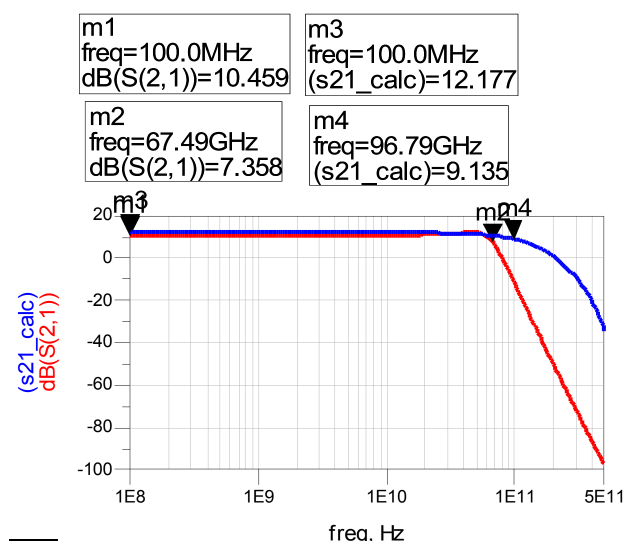
Gate: $V_g e^{-A_g \cdot \frac{3}{2}}$

Drain, at output: $g_m V_g e^{-A_g \frac{3}{2}} e^{-A_d \frac{1}{2}}$

Amplifier output: $g_m V_g \frac{Z_0}{2} \left(e^{-A_g/2} e^{-3A_D/2} + e^{-A_g \cdot \frac{3}{2}} e^{-A_d \frac{1}{2}} \right)$

Problem #6





Eqn Ag = 3.57e-23 * freq**2

Eqn Ad = 0.357

Eqn s21_calc = 20*log(.1143*50/2*(exp(-Ag/2)*exp(-Ad*3/2) + exp(-Ag*3/2)*exp(-Ad/2)))

The low frequency gain has about 1.7dB difference between calculation and simulation, which is expected as seen in the last problem where there is a 1.5 dB disparity in the output loss. The simulated cutoff frequency is 67.5GHz, very close to the expected 70GHz. However, the calculated cutoff is 97GHz, a significant disparity which is probably due to the inaccuracies in modeling the gate and drain losses.

(7) Keep transistor the same but set $R_i = R_{DS} = 0$ (no loss!)

Set $N = 10$.

Increase f_c to double (140 GHz) on output line, keeping same impedance.

$f_c = \frac{1}{\pi \sqrt{L_{new} C_{new}}}$, to double f_c, set $L_{new} = \frac{1}{2} L_{old} = 113.5\,pH$

$C_{new} = \frac{1}{2} C_{old} = 45.5\,fF$

Now $f_c = 140\,GHz$

and $Z_0 = \sqrt{\frac{L}{C}} = 50\,\Omega$ still.

If gate delays were same as drain delays we would get an output $= g_m V_g \frac{Z_0}{2} \times N$, since there is no loss.

But by changing output line now $T_D = \sqrt{LC} = 2.27\,ps$

and $T_g = 4.55\,ps$

$\Delta T = 2.28\,ps$

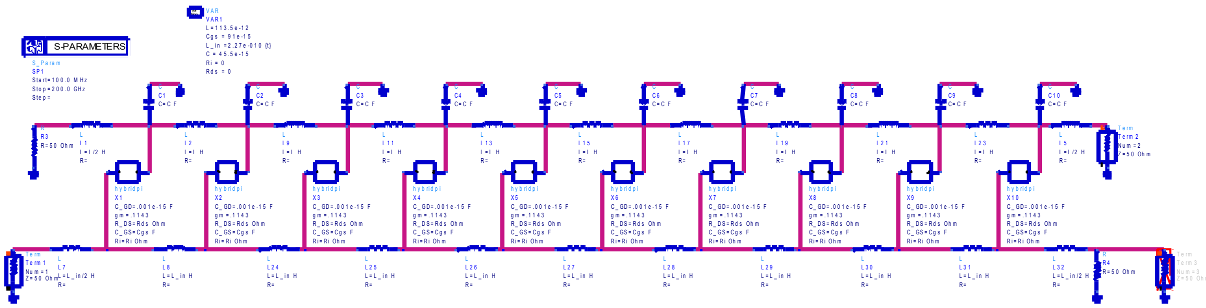
This mismatch causes a gain rolloff

$$V_o = g_m V_g \frac{Z_0}{2} \left(e^{-j\omega 9 \Delta T} + e^{-j\omega 8 \Delta T} + \cdots + 1 \right)$$

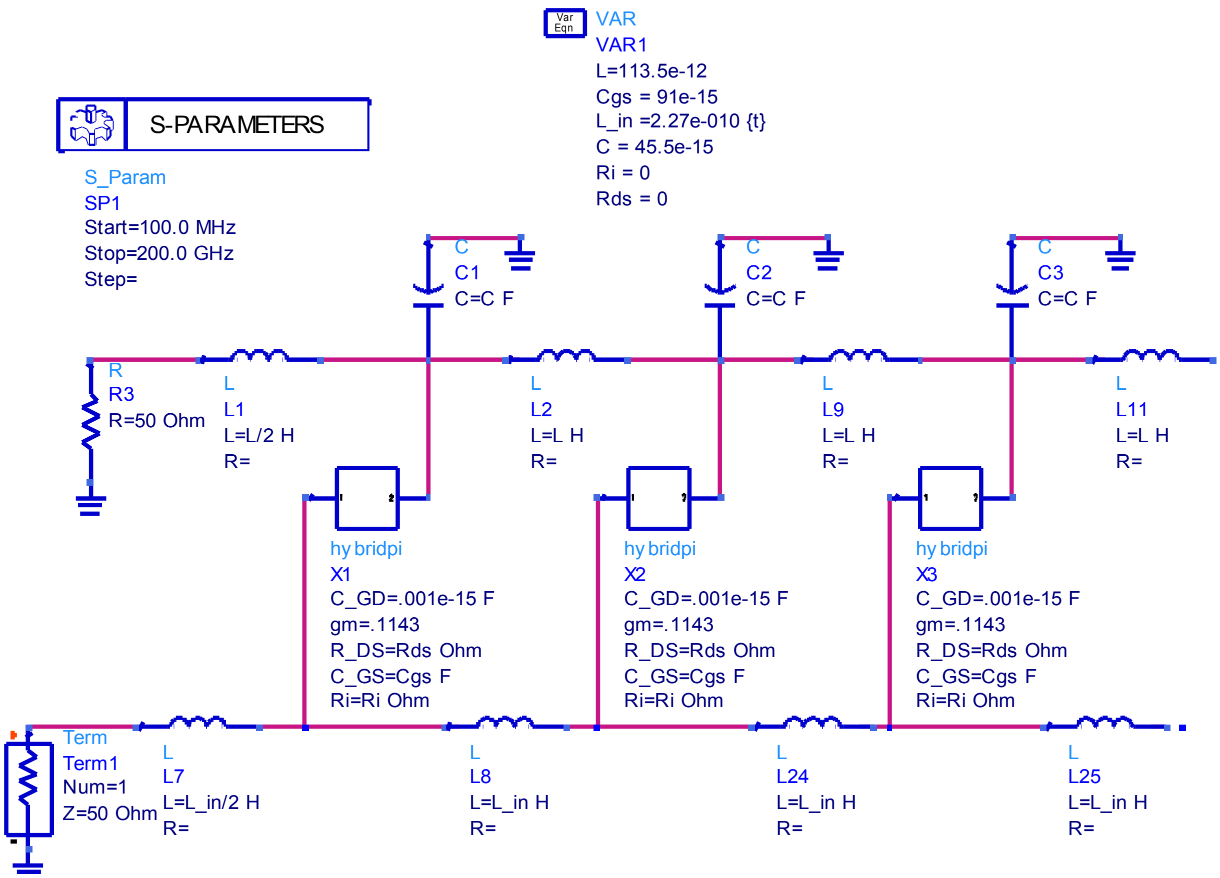
$$V_o = g_m V_g \frac{Z_0}{2} \sum_{l=0}^{9} e^{-j\omega l \cdot \Delta T}$$

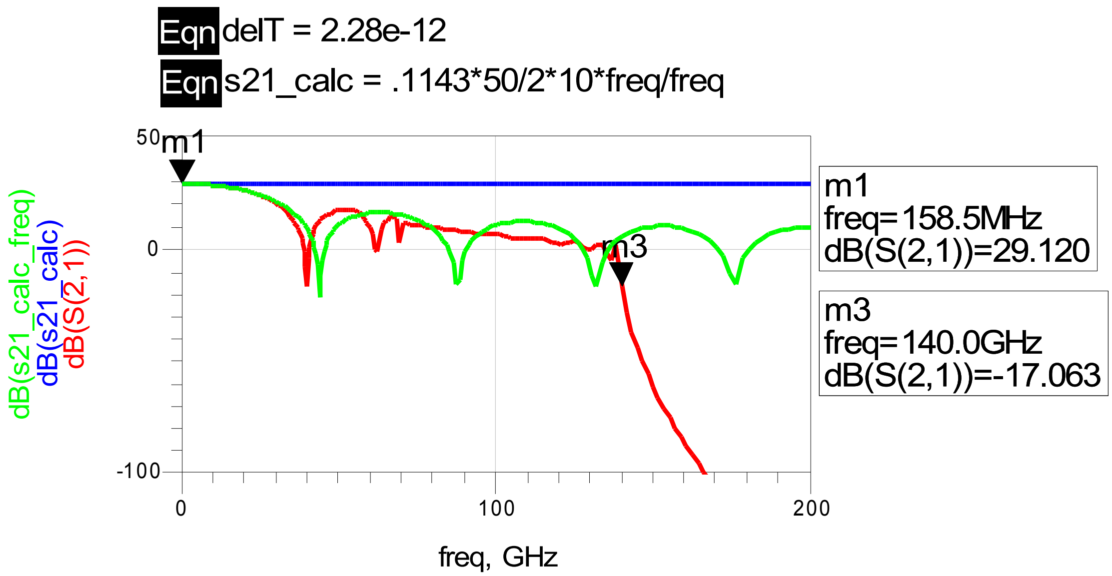
Simulation shows this accounts for most of the gain rolloff.

Problem #7



Zoomed in partial view:





Note: s21_calc_freq does not fit in this screenshot, see the written portion of solutions.

The low frequency gain calculation obviously agrees well with the low frequency gain of this amplifier. The frequency dependent portion which accounts for the attenuation which takes place as a result of mismatch in the time delay between input and output transmission lines is fairly consistent up to the output transmission line cutoff frequency of 140GHz, because that cutoff frequency is not modeled in our calculations.