① See HW5 #1 for same problem

Find MAG & H21 for unilateral device model
Then calculate f_T, f_{max}

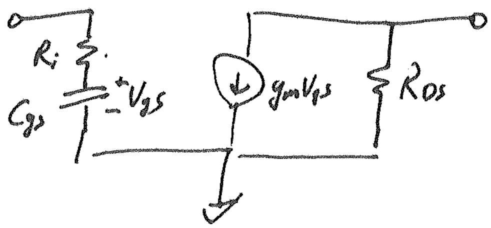
$g_m = 1 \frac{mS}{\mu m} W_g$

$R_i = 1/g_m = \frac{1000\Omega}{\mu m \cdot W_g}$

$C_{gd} = 0$

$C_{gs} = 0.5\, fF/\mu m \cdot W_g$

$G_{DS} = 0.1 \frac{mS}{\mu m} W_g \rightarrow R_{DS} = 10 \frac{k\Omega}{\mu m \cdot W_g}$



As we have already found, $MAG = \frac{g_m^2 R_{DS}}{4\omega^2 C_{gs}^2 R_i}$

$$= \frac{(1 \frac{mS}{\mu m} W_g)(\frac{10 \mu m}{mS\, W_g})}{4\omega^2 (0.5 \frac{fF}{\mu m} W_g)^2 (\frac{1 \mu m}{mS \cdot W_g})}$$

All W_g's cancel!

$$= \frac{1}{(2\pi)^2 f^2 10^{-26}}$$

@ f_{max} MAG = 1

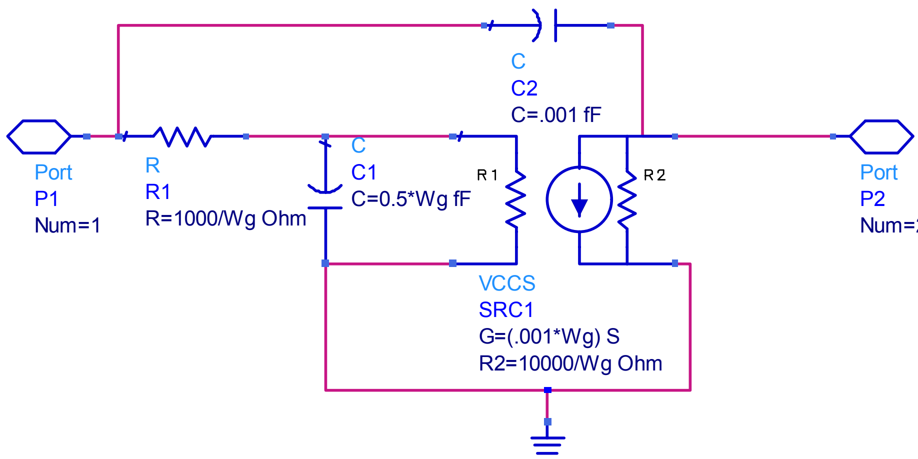
$$f_{max} = \sqrt{\frac{1}{(2\pi)^2 10^{-26}}} = \boxed{503\ GHz = f_{max}}$$

$H_{21} = \frac{g_m}{j\omega C_{gs}} = 1$ @ f_T

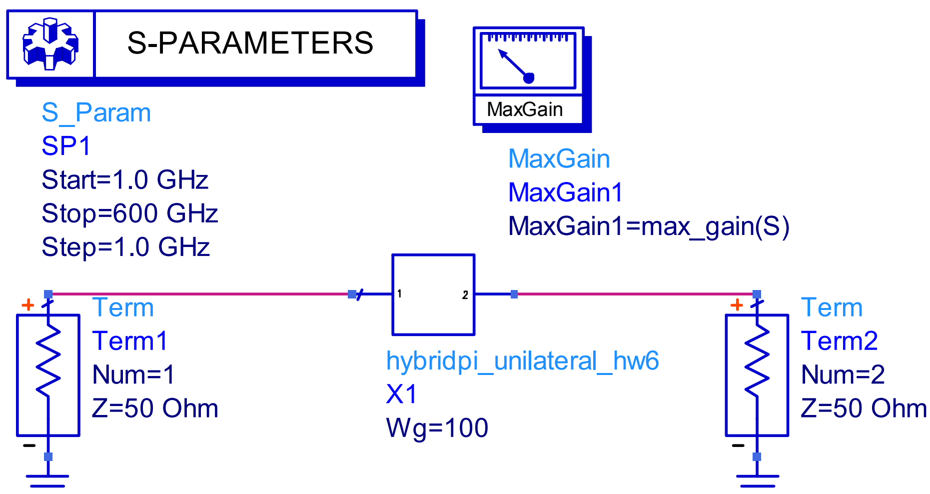
$$f_T = \frac{g_m}{2\pi C_{gs}} = \frac{1 mS/\mu m \cdot W_g}{2\pi \cdot 0.5 \frac{fF}{\mu m} W_g} = \frac{.001}{\pi \times 10^{-15}} = \boxed{318\ GHz = f_T}$$

Problem #1

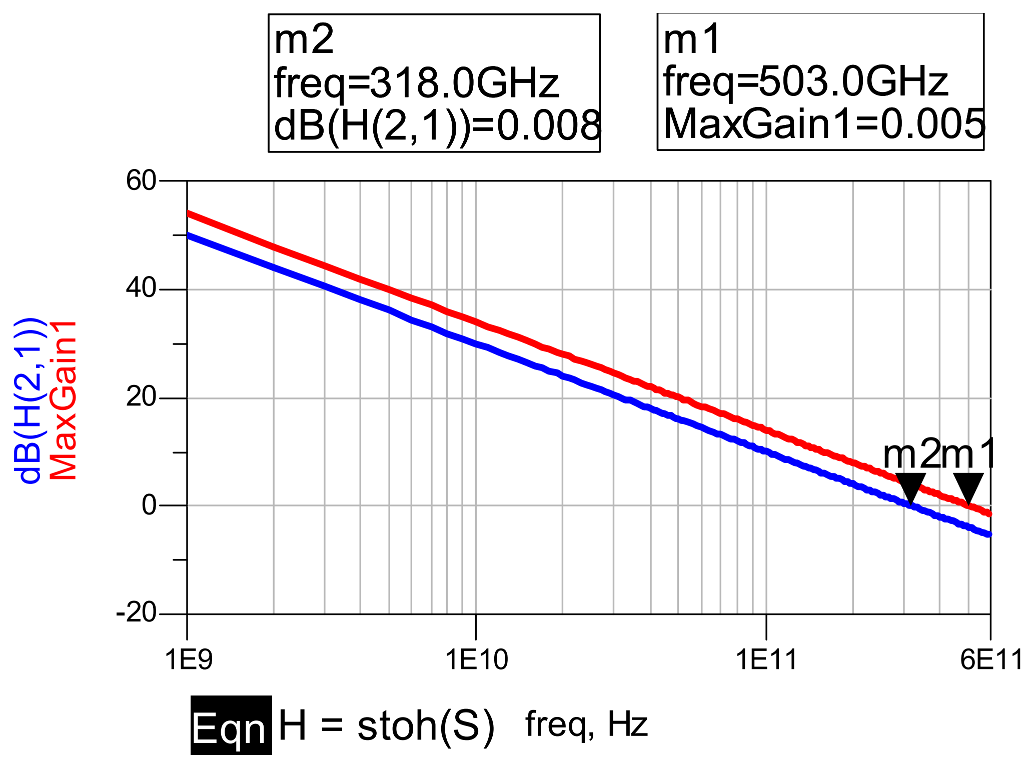
Unilateral Model:



Schematic:



Results:



② For same unilateral device, set $W_g = 100\,\mu m$.

Find Insertion Pwr gain, G_P, G_A, MAG.

$g_m = 100\,mS$

$R_i = 10\,\Omega$

$C_{gs} = 50\,fF$

$G_{DS} = 10\,mS$, $R_{DS} = 100\,\Omega$

<u>Insertion gain</u> = gain with $50\,\Omega$ source & load impedance

- $G_i = \| S_{21} \|^2$
- $S_{21} = \dfrac{2 V_{out}}{V_{gen}}$
- $V_{out} = g_m V_{gs} (R_{DS} \| Z_0)$
- $V_{gs} = \dfrac{1/j\omega C_{gs}}{R_i + Z_0 + \frac{1}{j\omega C_{gs}}} \cdot V_{gen} = \dfrac{V_{gen}}{1 + j\omega C_{gs}(R_i + Z_0)}$
- $V_{gen} = V_{gs}(1 + j\omega C_{gs}(R_i + Z_0))$

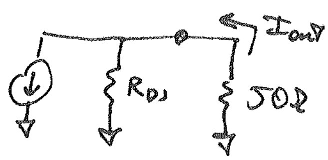
$$S_{21} = \frac{2 g_m V_{gs} R_{DS} \| Z_0}{1 + j\omega C_{gs}(R_i + Z_0)} = \frac{2(0.1)(33.3)}{1 + j2\pi f(50e{-15})(60)} = \frac{6.666}{1 + jf(18.85ps)}$$

$S_{21}(100GHz) = 3.12$

$10 \log(3.12) = \boxed{9.88dB = G_i}$

↳ because it's power gain

<u>Operating Gain (G_P)</u> = gain with input matched



As in ① $P_{in} = \omega^2 C_{gs}^2 V_{gs}^2 R_i$

$P_{out} = I_{out}^2 R_L = \left(g_m V_{gs} \underbrace{\frac{R_{DS}}{R_{DS} + Z_0}}_{\text{current divider}}\right)^2$

$$G_P = \frac{P_{out}}{P_{in}} = \frac{g_m^2 V_{gs}^2 R_{DS}^2 Z_0}{(R_{DS} + Z_0)^2 \omega^2 C_{gs}^2 V_{gs}^2 R_i} = \left(\frac{g_m R_{DS}}{(R_{DS} + Z_0)\omega C_{gs}}\right)^2 \times \frac{Z_0}{R_i}$$

$$= \left(\frac{.1 \times 100}{(150)(2\pi 100e9)(50e{-15})}\right)^2 \frac{50}{10}$$

$$= 22.5$$

$$\boxed{G_P = 13.52dB}$$

2) -cont.-

Available Pwr Gain G_a = Gain w/ output matched = $\frac{P_{av,amp}}{P_{av,gen}}$

- $P_{out} = \frac{1}{4} g_m^2 V_{gs}^2 R_{DS}$ (same as MAG's P_{out})

- $V_{gs} = I_{in}/j\omega C$, • $I_{in} = \frac{V_{gen}}{Z_{gen} + Z_{in}} = \frac{V_{gen}}{Z_0 + R_i + 1/j\omega C}$

$$V_{gs} = \frac{1}{j\omega C} \cdot \frac{V_{gen}}{Z_0 + R_i + 1/j\omega C} = \frac{V_{gen}}{1 + j\omega C(R_i + Z_0)}$$

so, $$P_{out} = \frac{\frac{1}{4} g_m^2 V_{gen}^2 R_{DS}}{[1 + j\omega C(R_i + Z_0)]^2}$$

$$P_{in} = P_{av,gen} = \frac{1}{4} \frac{V_{gen}^2}{Z_0}$$

$$G_a = \frac{P_{out}}{P_{in}} = \frac{\frac{1}{4} g_m^2 V_{gen}^2 R_{DS} Z_0}{\frac{1}{4} V_{gen}^2 (1 + j\omega C(R_i + Z_0))^2} = \frac{g_m^2 Z_0 R_{DS}}{[1 + j\omega C(Z_0 + R_i)]^2}$$

Evaluate @100 GHz, $|G_a| = 10.986$ $\qquad 10 \cdot \log(10.986) = 10.4$

$G_a = 10.4\text{dB}$

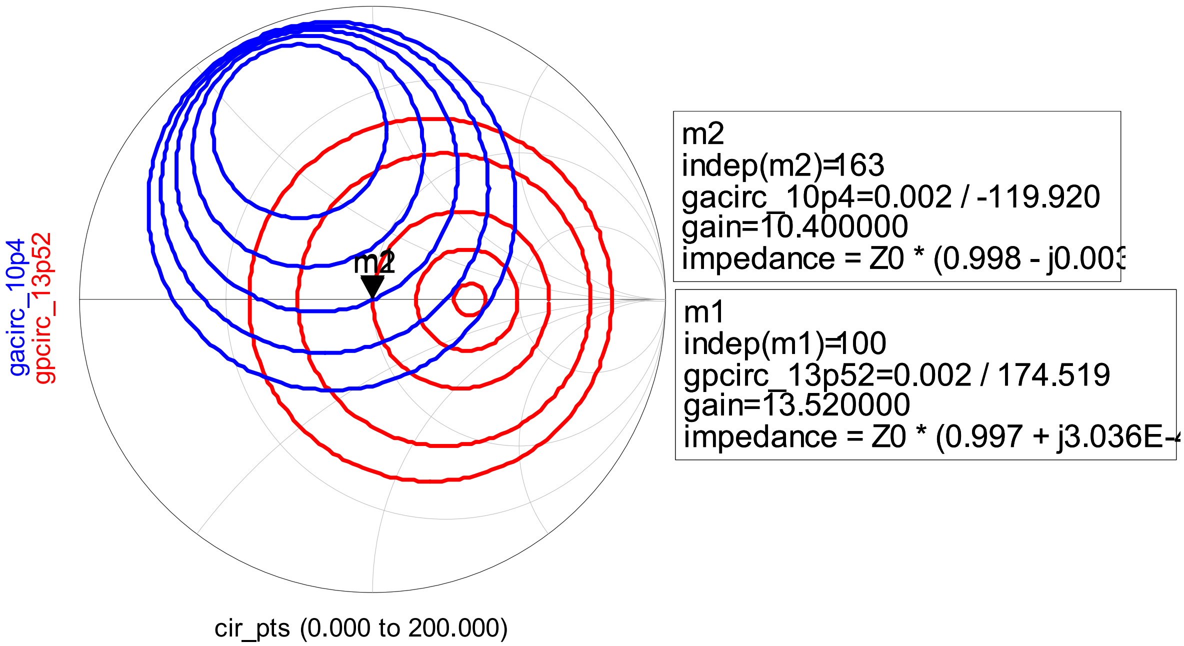
MAG

$$MAG = \left. \frac{g_m^2 R_{DS}}{4\omega^2 C_{gs}^2 R_i} \right|_{f=100e9} = \frac{(0.1)^2 100}{4(2\pi 100e9)^2 (50e-15)^2 10} = 25.3$$

$MAG = 14\text{dB}$

Note: All of these gains can be found with S-parameters and Γ_s, Γ_L in the general 2-port gain equations, depends only on your preference.

Problem #2



Eqn gpcirc_13p52 = gp_circle(S,13.52 + {-2, -1, 0, 0.35, 0.5},200)

Eqn gacirc_10p4 = ga_circle(S,10.4 + {-2, -1, 0, 1, 2},200)

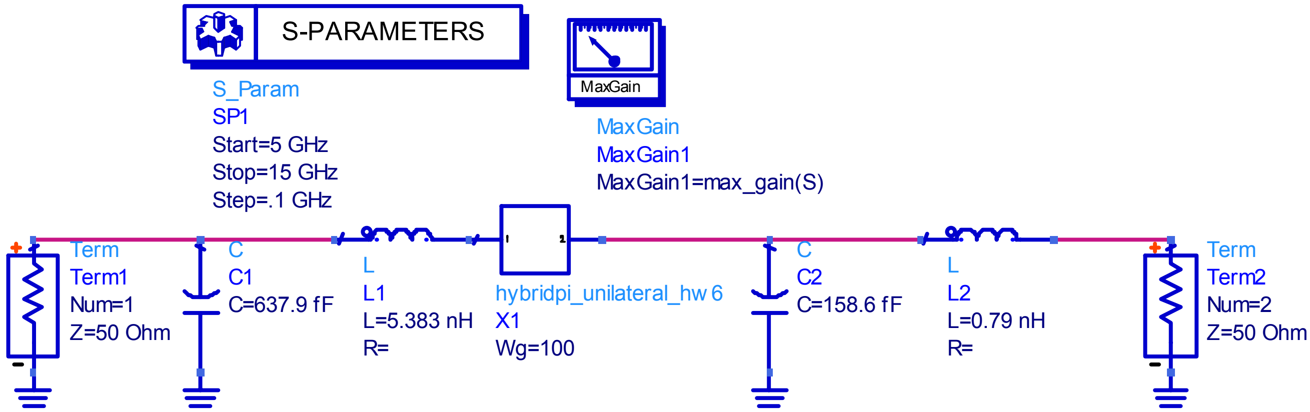
freq	db(S(2,1))	10*log(mag(S(2,1))**2)	MaxGain1
100.0 GHz	9.895	9.895	14.036

In the above displayed Ga and Gp circles, several gain values are plotted, above and below our calculated values for 50 ohm source and load impedance. We can see that the values of Ga and Gp determined in calculation occur for 50 ohm source and load impedance, respectively, because they pass through the center of the smith chart.

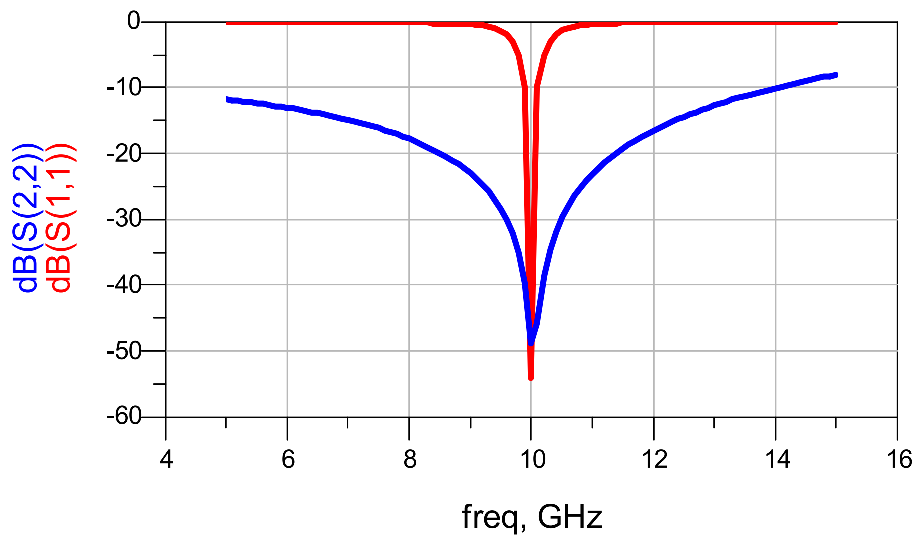
The insertion gain also matches well. This can be shown in two ways, as 10*log(mag(S21)^2), using 10*log because it is a power gain, or equivalently db(S21), which is 20*log(S21). These are mathematically identical. Max gain is also consistent with our calculation.

Problem #3

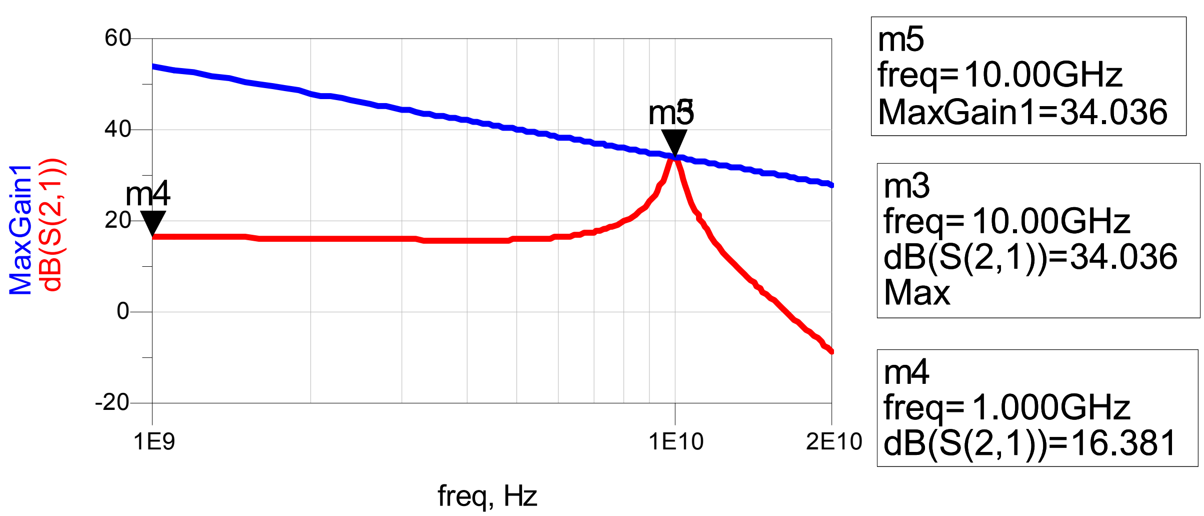
Using ADS tuning, an LC matching network can be designed:



S11 and S22



Gain:

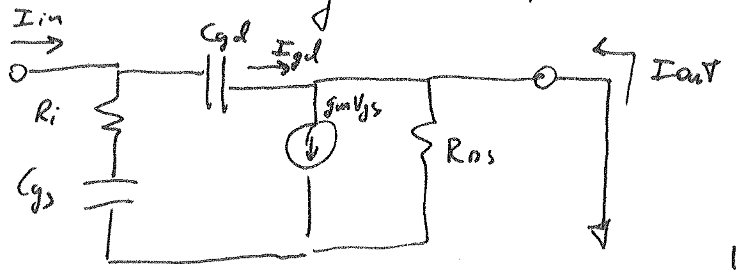


We can see that the gain when well-matched is equal to the maximum gain of the transistor, because the definition of maximum gain is when the input and output are both conjugate matched. The matching networks create an impedance equal to the conjugate of the input and output impedances.

④ Take bilateral device w/ $W_g = 100\mu m$, find f_T, f_{max}

$C_{gd} = 0.1 \frac{fF}{\mu m} \cdot W_g$ (other parameters same)

Start with finding H21 & f_T:



$$V_{gs} = \overbrace{I_{in}\left[(R_i + 1/j\omega C_{gs}) \,||\, 1/j\omega C_{gd}\right]}^{V_{in}} \cdot \overbrace{\frac{1/j\omega C_{gs}}{R_i + 1/j\omega C_{gs}}}^{\text{Voltage divider}}$$

$$= I_{in} \cdot \frac{(R_i + 1/sC_{gs})(1/sC_{gd})}{R_i + 1/sC_{gs} + 1/sC_{gd}} \cdot \frac{1}{1 + sC_{gs}R_i}$$

$$= I_{in} \cdot \frac{sC_{gs}(R_i + 1/sC_{gs})}{s^2 R_i C_{gs} C_{gd} + s(C_{gs} + C_{gd})} \cdot \frac{1}{1 + sC_{gs}R_i}$$

$$V_{gs} = \frac{I_{in}}{s^2 R_i C_{gs} C_{gd} + s(C_{gs} + C_{gd})}$$

$$I_{out} = g_m V_{gs} - I_{gd}, \qquad I_{gd} = I_{in} \cdot \overbrace{\left[(R_i + 1/j\omega C_{gs}) \,||\, 1/j\omega C_{gd}\right] \cdot \frac{1}{1/j\omega C_{gd}}}^{\text{Current Divider}}$$

$$= \frac{I_{in}(1 + sC_{gs})sC_{gd}}{s^2 R_i C_{gs} C_{gd} + s(C_{gs} + C_{gd})}$$

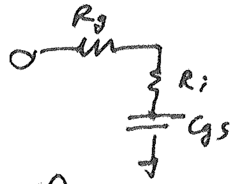
$$I_{out} = \frac{g_m - (1 + sC_{gs})sC_{gd}}{s^2 R_i C_{gs} C_{gd} + s(C_{gs} + C_{gd})} \cdot I_{in}$$

$$H_{21} = \frac{g_m - (1 + sC_{gs})sC_{gd}}{s^2 R_i C_{gs} C_{gd} + s(C_{gs} + C_{gd})} \quad \text{(matches ADS)}$$

$|H_{21}| = 1$ @ f_T → Challenging to solve by hand, solve with computer:

$\boxed{f_T = 318\ \text{GHz}}$ Note: same as unilateral case because R_i is not in series w/ both C_{gs} + C_{gd}

4) -cont.-



Find f_{max}: Using eqn from Rodwell's notes

$$f_{max} \cong \frac{f_T}{2\sqrt{(R_i + \cancel{R_s} + \cancel{R_g})\,G_{DS} + 2\pi f_T \cancel{R_g} C_{gd}}}$$

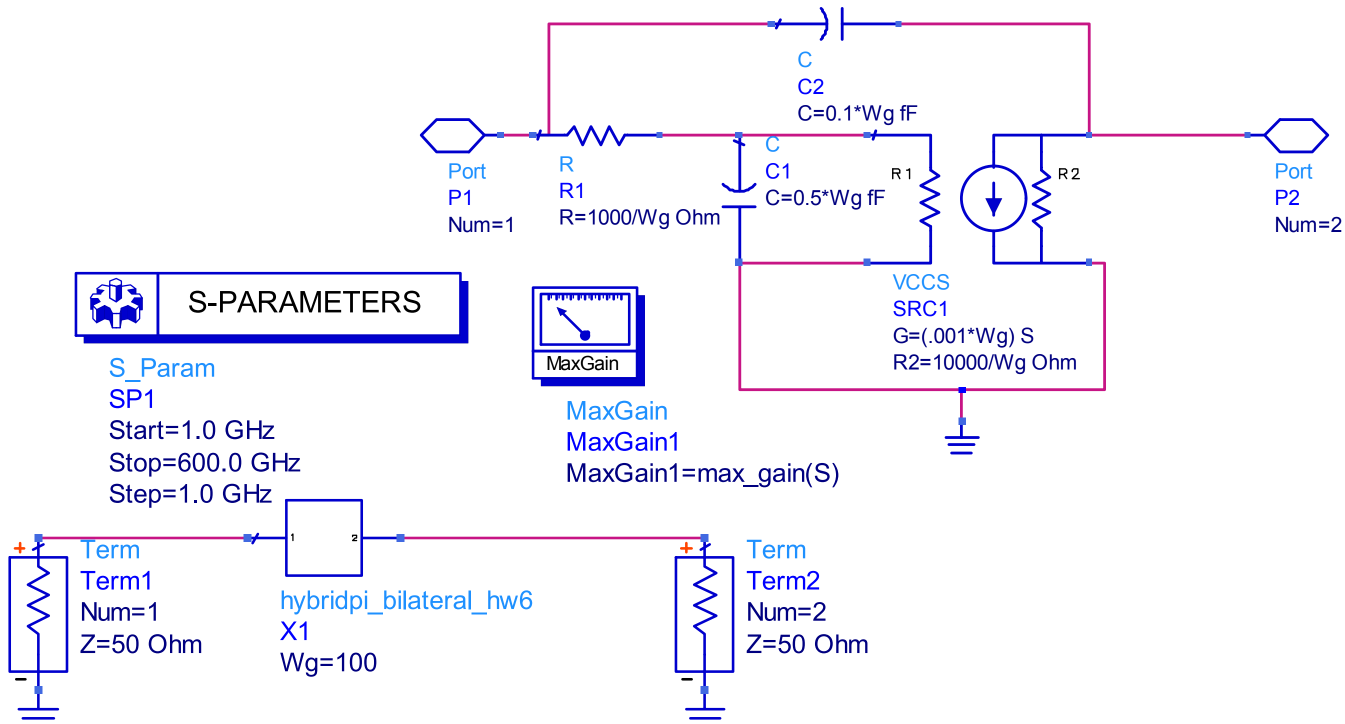
once again, because $R_g = 0$ we get same as unilateral case

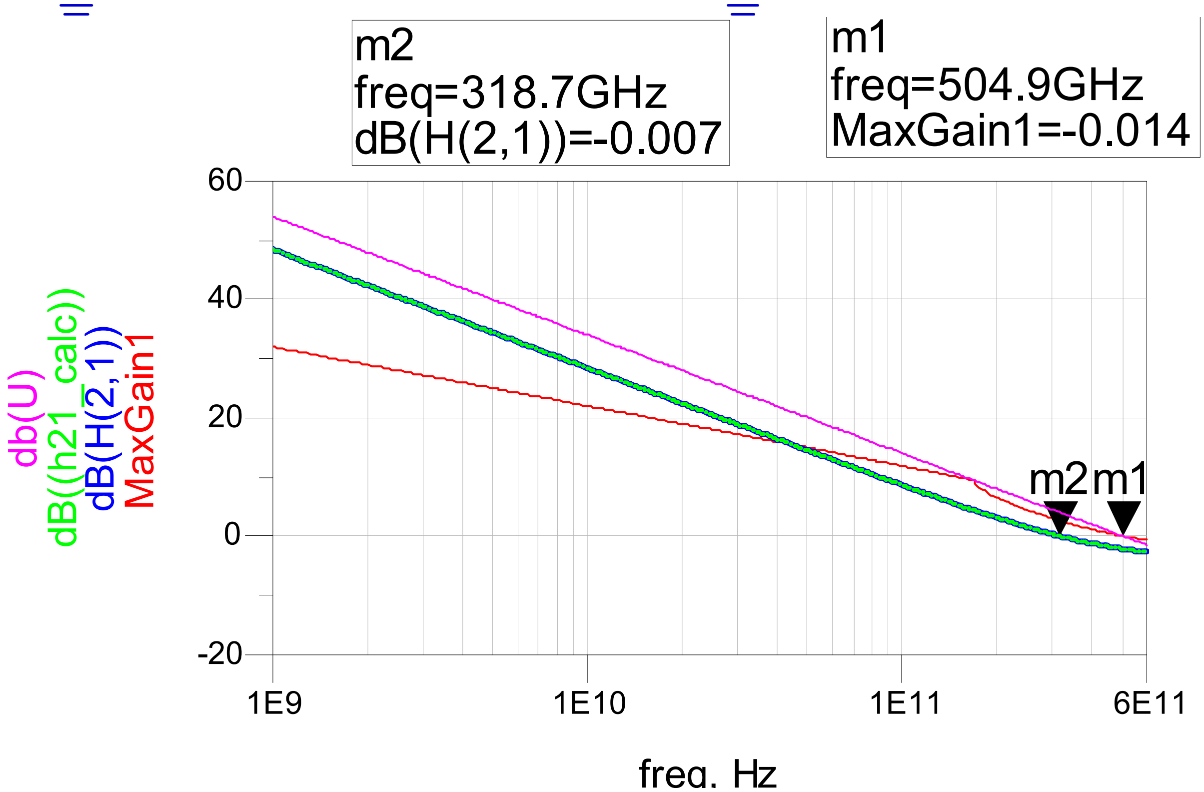
$$f_{max} = \frac{218\text{ GHz}}{2\sqrt{10 \cdot \frac{1}{100}}}$$

$$\boxed{f_{max} = 503\text{ GHz}}$$

(5)
(6) } see simulations & notes
(7)

<u>Problem #4</u>





Eqn H = stoh(S)

Eqn cgd = .1e-15*wg

Eqn wg=100

Eqn gm = .001*wg

Eqn h21_calc = (gm - (1 + s*cgs*ri)*s*cgd)/(s**2*ri*cgs*cgd + s*(cgs + cgd))

Eqn s=j*2*pi*freq

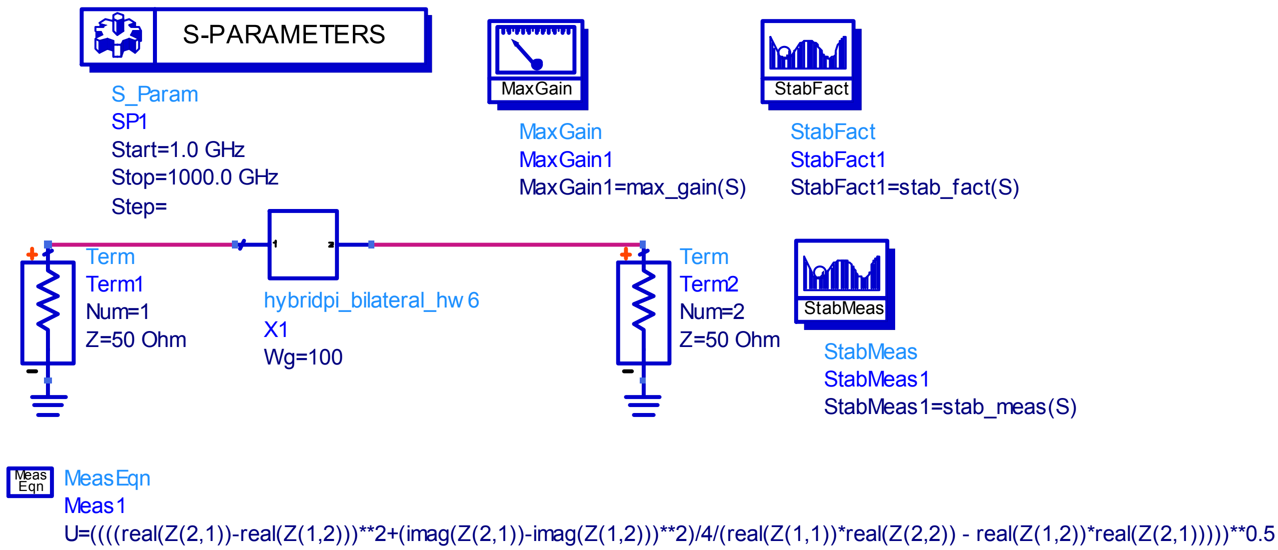
Eqn cgs = .5e-15*wg

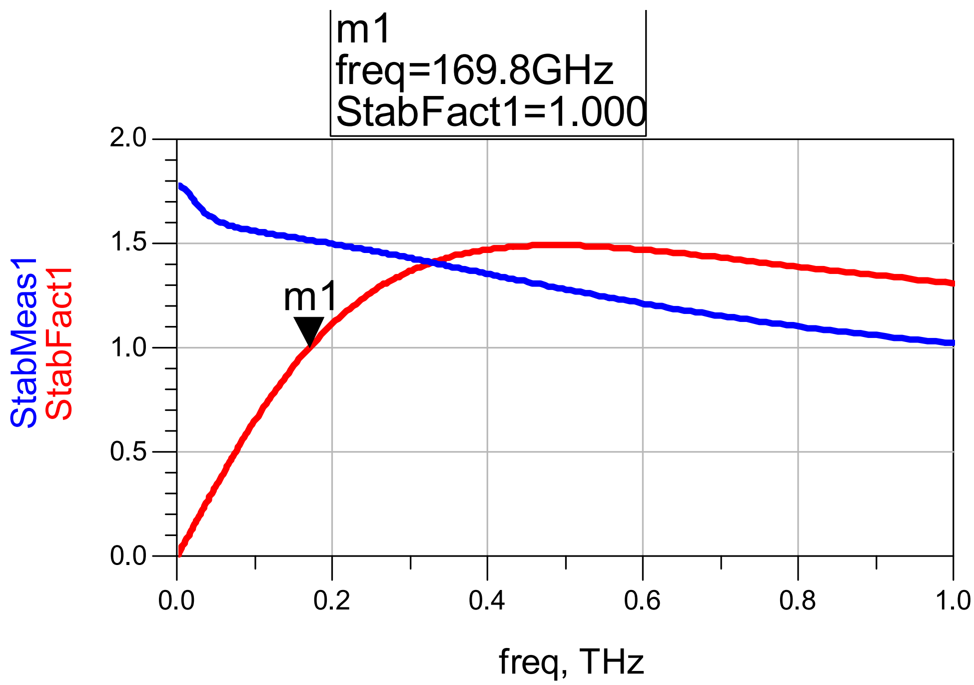
Eqn ri = 1000/wg

As we can see above, the calculated f_{max} and f_t match well with calculations. It is important to note that these values are the same as for the unilateral case, because in this case there is no R_g in series with both C_{GS} and C_{GD}.

Problem #5

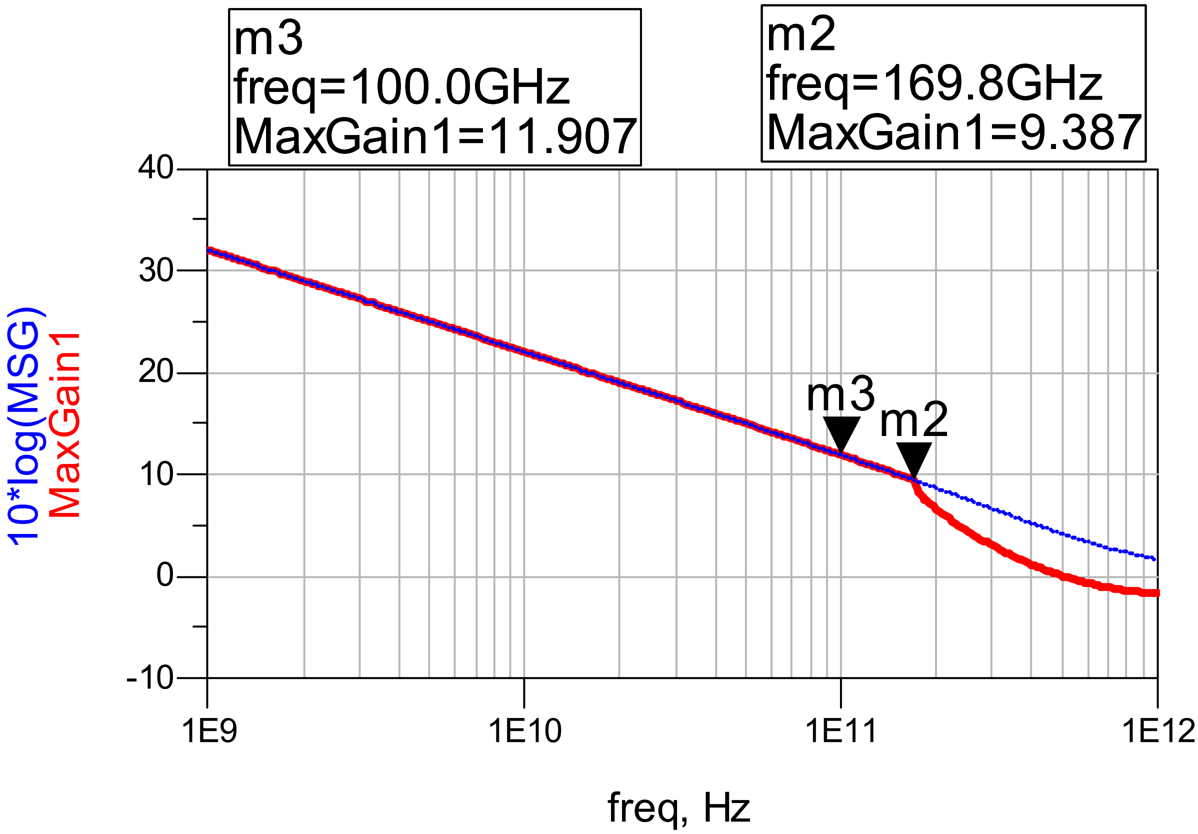
Plot stability factor K and B1 vs. frequency. It is necessary and sufficient for unconditional stability that K>1 and B1>0. In ADS k=stab_fact(S) and B1=stab_meas(S).





We can see that from DC-170GHz the transistor is potentially unstable, and it is unconditionally stable at all higher frequencies.

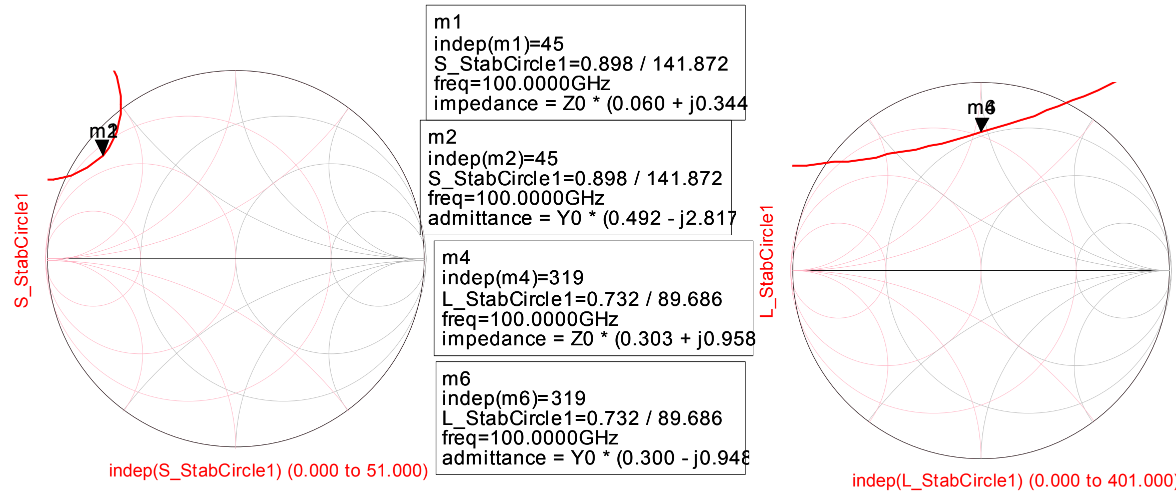
At a 100GHz we calculate the maximum stable gain, mag(S21)/mag(S12) and plot it in ADS. We also plot the ADS max_gain() function, which will automatically switch from MAG to MSG when the transistor becomes unstable, and it matches with our calculation. The MSG at 100GHz is 11.9dB.



Eqn MSG = mag(S(2,1))/mag(S(1,2))

Problem #6

Stability circles at 100GHz:



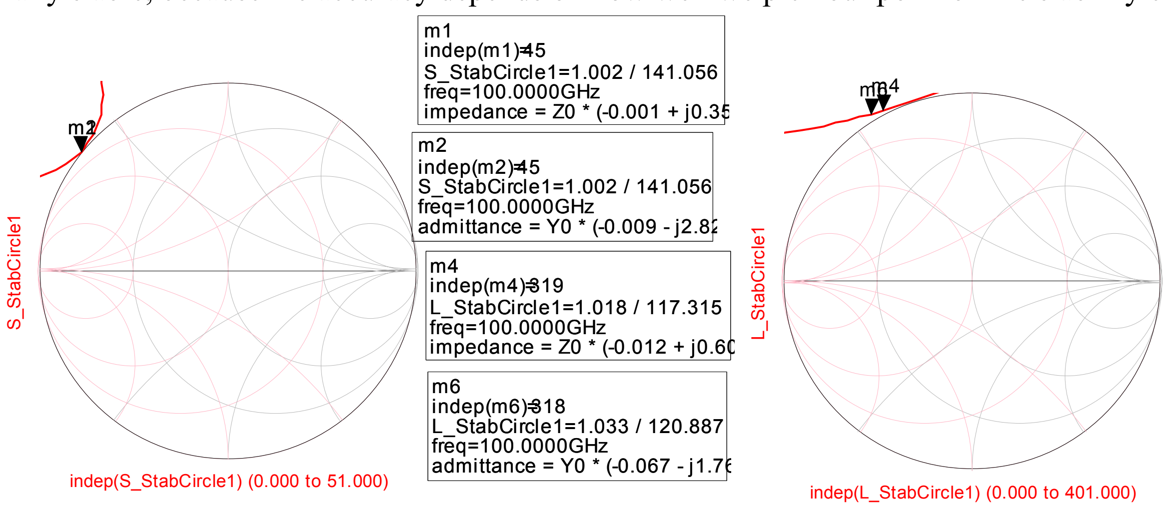
Input:

- Series R = .06*50 = 3Ω
- Shunt R = 50/.492 = 101.6Ω

Output:

- Series R = .303*50 = 15.15Ω
- Shunt R = 50/.3 = 166.67Ω

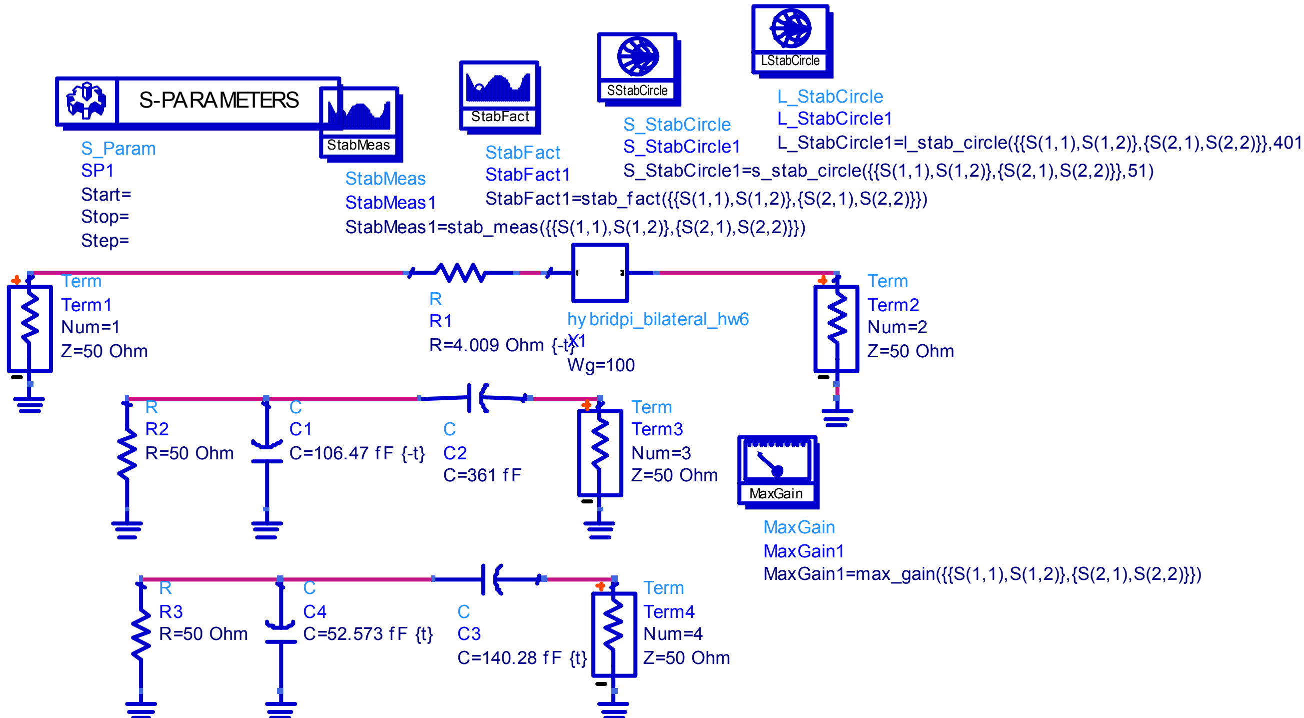
The first of these, series 3.1Ω on input, is shown. This value was used because 3Ω was very slightly not unconditionally stable, because the accuracy depends on how well we pick our point on the stability circle.



freq	StabFact1	StabMeas1
100.0 GHz	1.004	1.433

Problem #7

As we found, MSG at 100GHz is 11.9dB. Use a series input resistor to stabilize with a value such that stabilized MAG = 9.9dB. This was found to be R=4Ω.



freq	MaxGain1	StabFact1	StabMeas1
100.0 GHz	9.907	1.108	1.402

Following are the gain circles used for calculating the matching network. The Ga circle specifies the allowable values of gamma_s (reflection into transistor input from the matching network) for a given gain, and Gp circle does the same for gamma_l. We select in this case gamma_s as the MSG we designed for above, and with that we calculate gamma_out = S22+S12*S21*gamma_s / (1-S11*gamma_S). Then to match we must choose gamma_l as the complex conjugate of gamma_out. This value of gamma_l determines gamma_in through a similar coupling equation, and because our transistor has been unconditionally stabilized, it is the conjugate of gamm_s, so the input is also matched.

Therefore the gain is the MSG that we specified and the input and output reflections are very small, for our design frequency of 100GHz.

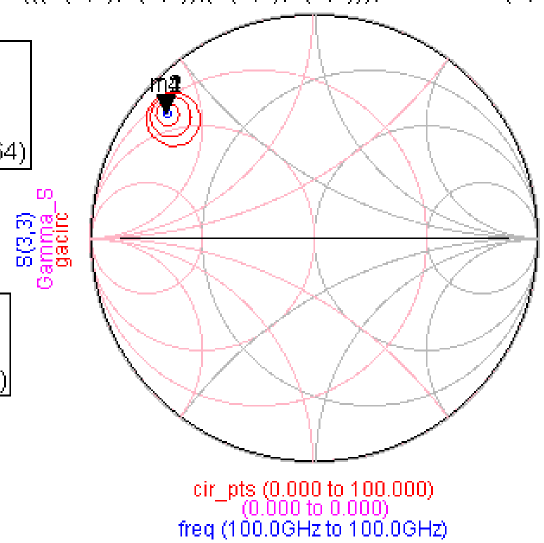
Eqn gpcirc=gp_circle({{S(1,1),S(1,2)},{S(2,1),S(2,2)}},MaxGain1 -{0,.1,.3,.5},100)

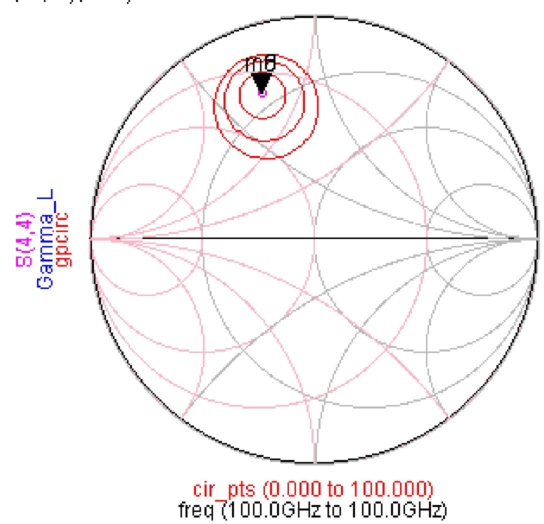
Eqn gacirc = ga_circle({{S(1,1),S(1,2)},{S(2,1),S(2,2)}},MaxGain1 -{0,.1,.3,.5},100)

m2
indep(m2)=100
gacirc=0.862 / 139.779
gain=9.907160
impedance = Z0 * (0.084 + j0.364)

m4
indep(m4)=0
Gamma_S=0.862 / 139.779
admittance = Y0 * (0.601 - j2.610)

m1
freq=100.0GHz
S(3,3)=0.862 / 139.788
admittance = Y0 * (0.601 - j2.610)





m3
indep(m3)=100
gpcirc=0.692 / 109.687
gain=9.907160
impedance = Z0 * (0.268 + j0.670)

m5
freq=100.0GHz
Gamma_L=0.692 / 109.687
impedance = Z0 * (0.268 + j0.670)

m6
freq=100.0GHz
S(4,4)=0.692 / 109.640
impedance = Z0 * (0.268 + j0.670)

Eqn Gamma_S=m2[0]

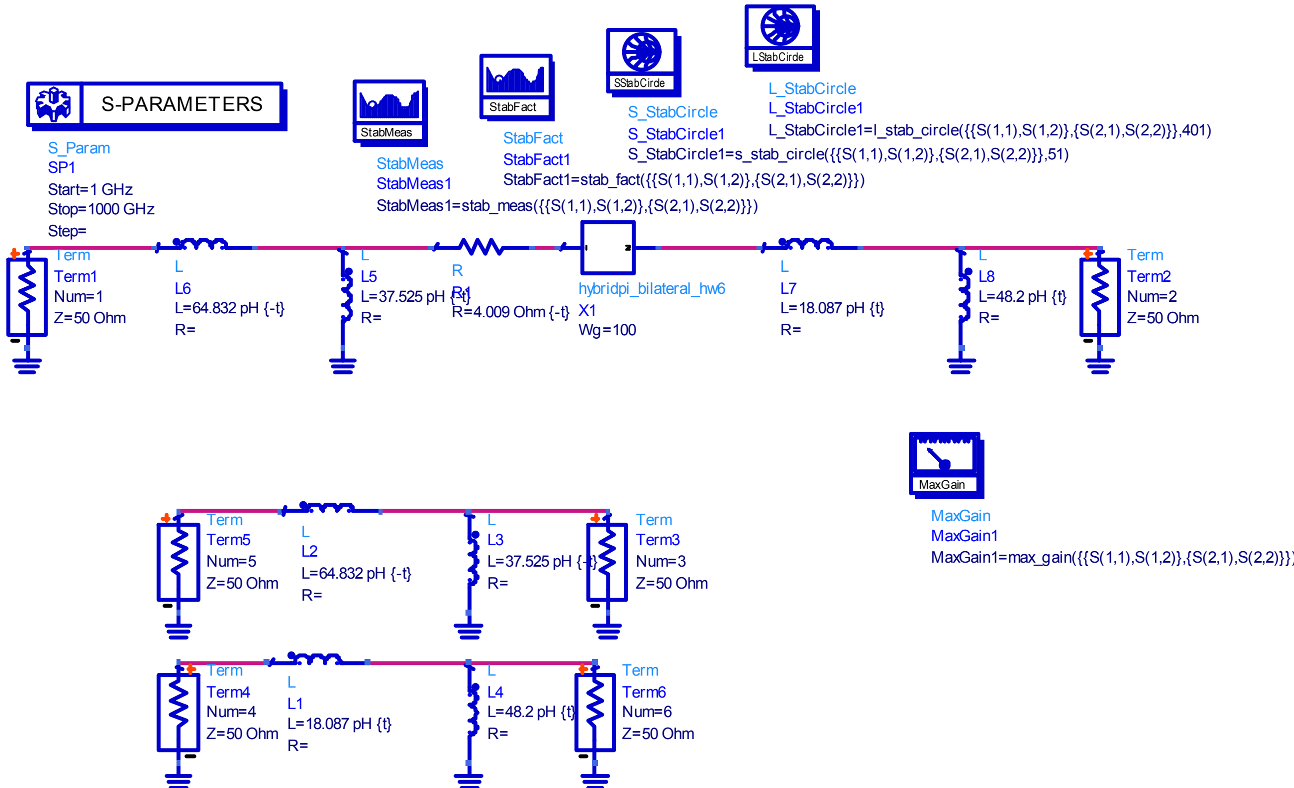
Eqn Gamma_in = conj(Gamma_S)

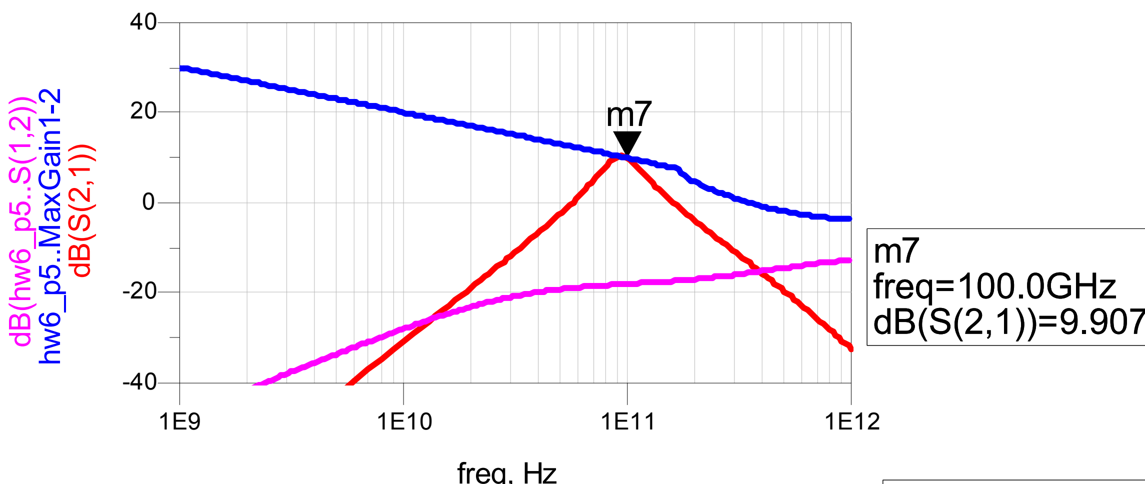
Eqn Gamma_L = conj(Gamma_out)

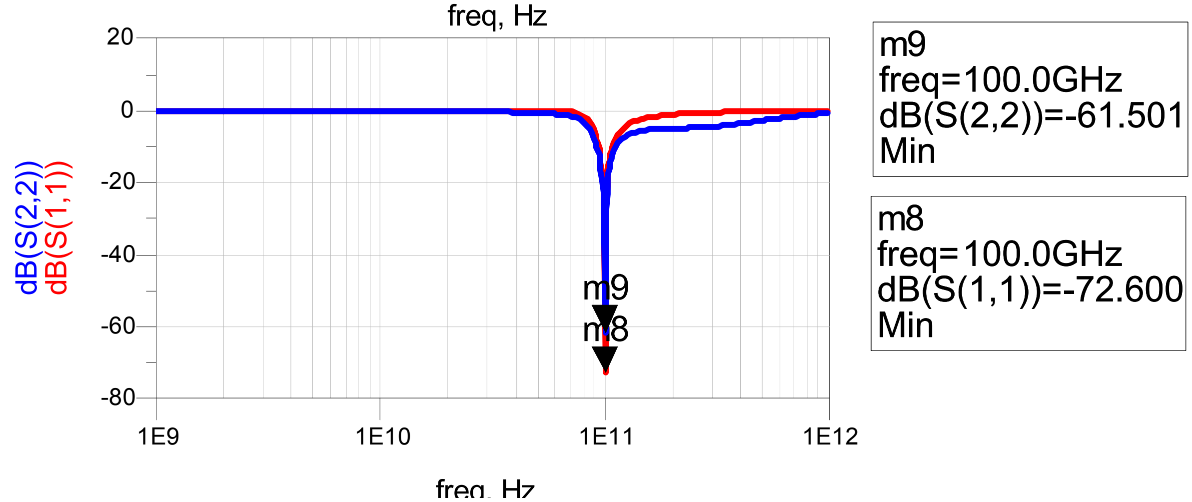
Eqn Gamma_out = S22+S12*S21*Gamma_S/(1-S11*Gamma_S)

Eqn Gamma_in2 = S11+S12*S21*Gamma_L/(1-S22*Gamma_L)

freq	Gamma_L	Gamma_out	Gamma_S	Gamma_in
100.0 GHz	0.692 / 109.687	0.692 / -109.687	0.862 / 139.779	0.862 / -139.779


S-PARAMETERS
S_Param
SP1
Start=1 GHz
Stop=1000 GHz
Step=
StabMeas
StabMeas
StabMeas1
StabMeas1=stab_meas({{S(1,1),S(1,2)},{S(2,1),S(2,2)}})
StabFact
StabFact
StabFact1
StabFact1=stab_fact({{S(1,1),S(1,2)},{S(2,1),S(2,2)}})
SStabCircle
S_StabCircle
S_StabCircle1
S_StabCircle1=s_stab_circle({{S(1,1),S(1,2)},{S(2,1),S(2,2)}},51)
LStabCircle
L_StabCircle
L_StabCircle1
L_StabCircle1=l_stab_circle({{S(1,1),S(1,2)},{S(2,1),S(2,2)}},401)
Term
Term1
Num=1
Z=50 Ohm
L
L6
L=64.832 pH {-t}
R=
L
L5
L=37.525 pH {t}
R=
R
R1
R=4.009 Ohm {-t}
hybridpi_bilateral_hw6
X1
Wg=100
L
L7
L=18.087 pH {t}
R=
L
L8
L=48.2 pH {t}
R=
Term
Term2
Num=2
Z=50 Ohm
MaxGain
MaxGain
MaxGain1
MaxGain1=max_gain({{S(1,1),S(1,2)},{S(2,1),S(2,2)}})
Term
Term5
Num=5
Z=50 Ohm
L
L2
L=64.832 pH {-t}
R=
L
L3
L=37.525 pH {-t}
R=
Term
Term3
Num=3
Z=50 Ohm
Term
Term4
Num=4
Z=50 Ohm
L
L1
L=18.087 pH {t}
R=
L
L4
L=48.2 pH {t}
R=
Term
Term6
Num=6
Z=50 Ohm


40
20
0
-20
-40
1E9
1E10
1E11
1E12
freq, Hz
dB(hw6_p5..S(1,2))
hw6_p5..MaxGain1-2
dB(S(2,1))
m7
m7
freq=100.0GHz
dB(S(2,1))=9.907


20
0
-20
-40
-60
-80
1E9
1E10
1E11
1E12
freq, Hz
dB(S(2,2))
dB(S(1,1))
m9
m8
m9
freq=100.0GHz
dB(S(2,2))=-61.501
Min
m8
freq=100.0GHz
dB(S(1,1))=-72.600
Min