PSTAT 5E Name:
Summer 2009 Perm：

Midterm 2

Please follow the instructions carefully:

- You have 75 minutes to complete the exam, and you should attempt all questions. Be sure to fully explain your answers. Answers without work will receive no credit.
- Anyone found copying off of another student's exam or consulting their textbook or notes during the exam will be given an "F" for the course.
- You may use a calculator on the exam, but you may not use cell phone/pda/mp3 player as a calculator.
- If you have two answers to a single question, you must cross-out the answer you don't want graded and circle the answer you want graded. If you fail to do this, you will receive a zero on the problem.

Please read each question carefully and good luck!

1. [5 points] The length of time it takes students to complete a statistics examination is uniformly distributed and varies between 40 and 60 minutes.
(a). Find the mathematical expression for the probability density function. [1']
(b) Compute the probability that a student will take between 45 and 50 minutes to complete the examination. [2']
(b).What is the expected value and variance of the amount of time it takes a student to complete the examination? [2']

2. [5 points] John parks cars at a hotel. On the average, 6 cars will arrive in an hour. Assume that a driver's decision on whether to let John park the car does not depend upon any other person's decision. Define the random variable X to be the number of cars arriving in any hour period.
a. Compute the probability that exactly 5 cars will arrive in the next hour. [1']
b. Compute the probability that no more than 2 cars will arrive in the next hour. [2']
c. Compute the probability that exactly 3 cars will arrive in the next **two** hours. [2']

3. [10 points] Bob is taking general chemistry. His professor says that the scores on the final exam are normally distributed with a variance of 90.25. Bob's friend Jake got an 89 on the exam and was in the 85^{th} percentile. If Bob gets an 80, what percentile is he in?

4. [10 points] Mike's bowling scores are normally distributed. Last night Mike bowled 4 games at Zodo's and got the following scores:

224 211 228 237

(a) Find the mean and standard deviation for Mike's bowling scores last night. [2']
(b) Calculate a 95% Confidence Interval(CI) for Mike's true average bowling score.[3']
(c) If I tell you that Mike's true variance in bowling scores is 64, does this change the CI found in part (c)? If it does, please calculate the new CI. [3']
(d) Would your confidence interval be valid without the normal assumption about Mike's bowling scores? Explain. [2']

5. [10 points] A large financial firm is considering opening a franchise in San Diego and wants to estimate the mean household income for the area using a simple random sample of households. Based on information obtained from a study, the company assumes that the standard deviation of household incomes is $7,200. What is the least number of households which should be surveyed in order to obtain an estimate that is within $200 of the true mean household income with 95% confidence?

6.[5 points] A sample containing 100 people was asked what their favorite cereal type is.
(a) 60 people selected Cheerios, 30 people selected Rice Krispees, and 10 people selected Frosted Flakes. Construct a 90% confidence interval for the proportion of people who **do not** like Cheerios. [3']
(b) At the same confidence level 90%, what happens to the width of a confidence interval if the sample size is increased? [2']

7. [5 points] At 95% confidence, how large a sample should be taken to obtain a margin of error of 0.03 for the estimation of population proportion? (Assume the past data are not available for developing a planning value of p*.)

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Formulas:

$\bar{x} = \frac{\sum x}{n}$ $s^2 = \frac{\sum(x - \bar{x})^2}{n-1}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $P(A|B) = \frac{P(A \cap B)}{P(B)}$

If X is a Poisson(μ) RV: $P(X=k)=\frac{\mu^k e^{-\mu}}{k!}$, E(X)= μ, Var(X)= μ

If X is a Uniform(a, b) RV, then f(x) = $\frac{1}{b-a}$ $for\ a \le x \le b$

and μ = E[X] = $\frac{a+b}{2}$, $\sigma^2 = Var(X) = \frac{(b-a)^2}{12}$

Standardization: $Z = \frac{observed - mean}{st.dev.}$, $Z = \frac{X-\mu}{\sigma}$, $Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$, $Z = \frac{\bar{p}-p}{\sqrt{\frac{p(1-p)}{n}}}$

CLT: 1. If n ≥ 30 then $\bar{X}$ is Normal with mean = μ and st. dev. = $\frac{\sigma}{\sqrt{n}}$

2. If *np* ≥5 and *n(1-p)* ≥5 then $\bar{p}$ is Normal with mean = p and st. dev. = $\sqrt{\frac{p(1-p)}{n}}$

Fact: If you start with a Normal Population, then $\bar{X}$ is *always* Normally Distributed with mean = μ and st. dev. = $\frac{\sigma}{\sqrt{n}}$

100(1-α)% confidence Intervals:

$\bar{x} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ $\bar{x} \pm t_{n-1,\alpha/2} \frac{s}{\sqrt{n}}$ $\bar{p} \pm Z_{\alpha/2}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$

Minimum Sample Size:

$n = (\frac{Z_{\alpha/2}\ \sigma}{E})^2$ $n = \frac{(Z_{\alpha/2})^2\ p^*(1-p^*)}{E^2}$ where E is the margin of error