Midterm II Study Guide Solutions

1. Hint:
   X ~ Binomial Distribution with n=10, and p=0.32
   Calculate P(X=5) by binomial prob function (P220) and E(X)=np=3.2

2. Hint:
   μ= 9 claims each week , X=# of claims for one week, X~Poi(9)
   P(X=7) =9^7 *e^(-9)/7!
   P(X=0)= 9^0*e^(-9)/0!= e^(-9) (b/c 9^0=1 and 0!=1)

   μ= 9*3=27 claims for 3 weeks, X=# of claims for 3 weeks, X~Poi(27)
   P(X<4) =P(X=0)+P(X=1)+P(X=2)+P(X=3)=…

3. a) Note: <= means "less than or equal to", >= means "greater than or equal to"
   i) P(X>6) = P(Z>(6-7)/0.5) = 1-P(Z<=(6-7)/0.5)=1-P(Z<=-2.0)=1-0.0228=**0.9772**
   ii) P(7<=X<=8) = P(Z<=(8-7)/0.5) – P(Z<=(7-7)/0.5) = P(Z<=2.0) – P(Z<=0)
   = 0.9772 - 0.5 = **0.4772**
   iii) P(6.5<=X<=8.5) = P(Z<=(8.5-7)/0.5) – P(Z<=(6.5-7)/0.5)
   = P(Z<=3.0) – P(Z<=-1.0) = 0.9986 – 0.1587 = **0.84**

   b) P(X<7 | X>6) = P(X<7 and X>6)/P(X>6) = P(6<X<7)/P(X>6)
   P(6<X<7) = P(Z<(7-7)/0.5) – P(Z<(6-7)/0.5) = P(Z<0.0) – P(Z<=-2.0)
   = 0.5 – 0.0228 = 0.4772
   So, P(X<7 | X>6) = 0.4772/0.9772 = **0.4883**

4. Let variable X represent test score. 59 percentile corresponds to P(X<x) = 0.59, etc.
   So, using the Z-tables, 0.59 probability gives Z=0.23, 0.38 probability gives Z=-0.3
   Using Z-score formulas, Z = 0.23 = (63-u)/(stdev), Z = -0.30 = (53-u)/(stdev).
   So, using algebra, solve this system of 2 equations with 2 unknown variables, u (the mean), and stdev (the standard deviation).
   So, solve the 2 equations rewritten as: 0.23*(stdev)+u=63
   (-) -0.30*(stdev)+u=53

   0.53*(stdev) = 10, stdev = 18.87
   Now, plug the found stdev value back into any one of the 2 equations, to solve for u:
   I will use the first equation: 0.23*18.87+u=63, such that u=63-0.23*18.87=58.7

   Now, finding the percentile for X=85, which is P(X<85)=P(Z<(85-58.7)/18.87)
   =P(Z<1.396) = **0.9192**

5. n=51, Xbar=0.72, S^2=0.0936. So, S=sqrt(S^2)=sqrt(0.0936) = 0.306
   Finding a 95% confidence interval: t = 2.009, since using alpha value of 0.25 and degrees of freedom equals n-1=51-1=50. So, the 95% confidence interval is
   0.72 +/- 2.009*0.306/sqrt(51) = 0.72 +/- 0.086387 = **(0.634, 0.806)**

6. N=5246, u=1000, stdev=240, n=64. 1000-60=940, 1000+60=1060.
   P(not certified) = P(Xbar<940 U Xbar>1060) = P(Xbar<940) + P(Xbar>1060)
   = P(Z<(940-1000)/(240/sqrt(64))) + 1 – P(Z<(1060-1000)/(240/sqrt(64)))
   = P(Z<-2.0) + 1 – P(Z<2.0) = 0.0228 + 1 – 0.9772 = **0.0456**

7. P(go on) = P(X>160) = 1 – P(Z<(160-150)/30) = 1-P(Z<0.33) = 1-0.6293 = **0.3707**

8. n=81, Xbar=14.1, S=2.6. So, n-1=80 d.f., and thus t=1.99
   A. 95% confidence interval is: 14.1 +/- 1.99*2.6/sqrt(81) = 14.1 +/- 0.5749 = **(13.525, 14.675)**, since alpha is 0.05/2=0.025
   B. 90% confidence interval is: 14.1 +/- 2.369*2.6/sqrt(81) = 14.1 +/- 0.7624 = **(13.34, 14.86)**, since t=2.369 with alpha of 0.01/2=0.005 and d.f.=81-1=80
   C. If the level of confidence is increased while the sample size is held fixed then the width of the confidence interval **increases**.
   D. Yes, because the sample size is large enough.

9. n=400, x=40, sample p = 40/400=0.1. Also, Z=1.96 since alpha=0.05/2
   So, a 95% confidence interval is 0.1 +/- 1.96*sqrt(0.1*0.9/400)
   = 0.1 +/- 0.0294 = **(0.0706, 0.1294)**

10. First, sum up the 10 numbers and divide that sum by 10 to get the sample mean which is 34. Now, calculate the standard deviation of the 10 number sample, which is sqrt(52/(10-1)) = sqrt(5.7777) = 2.404. Now that the mean and standard deviation of the 10-number sample have been determined, proceed to find the 90% confidence interval, which is: 34 +/- 1.833*2.404/sqrt(10) = 34 +/- 1.3935 = **(32.61, 35.39)**, where t=1.833 is determined from the fact that this 10-number sample is not a population, and thus sigma (population stdev) is unknown. Therefore, need to use the stdev of this 10-number sample as an estimate for the population stdev, and thus need to find the t-value rather than the Z-value. d.f.=10-1=9 for this problem.