

Practice Midterm I Solutions

1.

- (1) **D**
- (2) **A**
- (3) **B**
- (4) **C**
- (5) **B**

2.

- a) mean = $(15+17+99+11+15+16)/6 = \mathbf{28.83}$
 median: first arrange numbers from least to greatest:
 11 15 15 16 17 99
 Now, take middle two numbers, average them.
 So, median = $(15+16)/2 = \mathbf{15.5}$
 mode: **15**, since it is the number occurring the most frequently
- b) variance = $[(15-28.83)^2+(17-28.83)^2+(99-28.83)^2+(11-28.83)^2+(15-28.83)^2+(16-28.83)^2]/(6-1) = 5,928.83/5 = \mathbf{1,185.77}$
- c) Looking at ordered data in increasing order:
 11 15 15 16 17 99, divide data into two sets: 11 15 15, and
 16 17 99 for data to the left and to the right of the median. The median of the first set, **15**, is the First Quartile, and the median of the second set, **17**, is the Third Quartile. The Interquartile Range, IQR, is equal to $17-15 = \mathbf{2}$.

3. Quarts SD	Frequency	Relative Frequency	Cumulative Frequency
0 --3	4	0.20	0.20
4 --7 5	5	0.25	0.45
8 --11 6	6	0.30	0.75
12 -- 15 3	2	0.10	0.85
16 --19 2	3	0.15	1.00
	----- 20	----- 1.00	

4.

- a) Let event T = event that Tom passes the test
 Let event D = event that Dick passes the test
 Since T and D are independent events, then $P(T \text{ and } D) = P(T)*P(D)$.
 $P(T) = 0.7$, $P(D) = 0.8$, $P(\text{not } T) = 1-0.7 = 0.3$, $P(\text{not } D) = 1-0.8 = 0.2$
 $P(\text{at most one of Tom or Dick passes the test}) = 1 - P(\text{not}(T \text{ or } D \text{ or neither})) = 1 - P(T \text{ and } D) = 1 - P(T)*P(D) = 1-(0.7*0.8) = \mathbf{0.44}$.
- b) Since T and D are independent events, then $P(T \text{ and } D) = P(T)*P(D)$, as well as $P(D \text{ and not } T) = P(D)*P(\text{not } T)$ and $P(\text{not } D \text{ and } T) = P(\text{not } D)*P(T)$.
 So, $P(D | \text{one of Tom or Dick passes}) = P(D \text{ and not } T) / [P(D \text{ and not } T) + P(\text{not } D \text{ and } T)]$

$$= [P(D)*P(\text{not}T)]/[P(D)*P(\text{not}T) + P(\text{not}D)*P(T)] = [0.8*0.3]/[0.8*0.3 + 0.2*0.7]$$

$$= \mathbf{0.6316}.$$

5.

a) First, it is imperative to fill in the table with the sums of each row, each column, and the sum of the rows' sums and the columns' sums, which should each be the same.

	<u>Men (M)</u>	<u>Women (W)</u>	TOTAL
<u>Promoted (P)</u>	288	36	324
<u>Not Promoted (NP)</u>	672	204	876
TOTAL	960	240	<u>1,200</u>

Now, $P(W \text{ and } NP) = 204/1,200 = \mathbf{0.17}$.

b) $P(NP | M) = 672/960 = \mathbf{0.7}$, which is NOT equal to $P(M | NP) = 672/876 = 0.767$.

c) Mutually Exclusive? $P \text{ and } W = 36$, which is nonempty. So, $P(P \text{ and } W) = 36/1,200$, which is NOT equal to 0. Thus, P and W are **NOT Mutually Exclusive**.
Independent? Determine whether $P(P | W) = P(P)$. $P(P) = 324/1,200 = 0.27$.
However, $P(P | W) = 36/240 = 0.15$, which is NOT equal to 0.27. So, $P(P | W)$ is not equal to $P(P)$. So, P and W are **NOT Independent**.

6.

a) Since A and B are Mutually Exclusive, then $P(A \text{ and } B) = 0.3 + 0.5 = \mathbf{0.8}$, since $P(A \text{ and } B) = 0$.

b) $P(D) = P(A \text{ and not}B) = P(A) = \mathbf{0.3}$. Draw a Venn-diagram of A and B to see why A and not B is equivalent to A , for mutually exclusive events A and B .

c) If A and B are independent events, then $P(A \text{ and } B) = P(A)*P(B)$.

However, $P(A \text{ and } B) = 0$, but $P(A)*P(B) = 0.3*0.5 = 0.15$, which is not equal to 0. So, since $P(A \text{ and } B)$ is not equal to $P(A)*P(B)$, then A and B are **NOT independent**.

7 (a) 0.2

(b) 0.5

(c) $E(X) = 0*(0.5) + 1*(0.3) + 2*(0.2) = 0.7$

8. a. $P(X = k) = f(k) = \binom{n}{k} (p)^k (1-p)^{n-k}$

$$n = 10, p = 0.3$$

$$f(3) = \frac{10!}{3!(10-3)!} (.30)^3 (1-.30)^{10-3}$$

$$f(3) = \frac{10(9)(8)}{3(2)(1)} (.30)^3 (1-.30)^7 = .2668$$

b. $P(x \geq 3) = 1 - f(0) - f(1) - f(2)$

$$f(0) = \frac{10!}{0!(10)!} (.30)^0 (1-.30)^{10} = .0282$$

$$f(1) = \frac{10!}{1!(9)!} (.30)^1 (1-.30)^9 = .1211$$

$$f(2) = \frac{10!}{2!(8)!} (.30)^2 (1-.30)^8 = .2335$$

$$P(x \geq 3) = 1 - .0282 - .1211 - .2335 = \mathbf{.6172}$$