Practice Midterm 2

Sections Covered: 5.5, Ch 6-8 (skip 6.3, 7.7)

Formulas:

$\bar{x} = \frac{\sum x}{n}$ $s^2 = \frac{\sum(x - \bar{x})^2}{n-1}$

If X is a Poisson(μ) RV: $P(X=k)=\frac{\mu^k e^{-\mu}}{k!}$, E(X)= μ, Var(X)= μ

If X is a Uniform(a, b), then f(x) = $\frac{1}{b-a}$ $for\ a \le x \le b$,

and μ = E[X] = $\frac{a+b}{2}$, $\sigma^2 = Var(X) = \frac{(b-a)^2}{12}$

Z-Transformation: $Z = \frac{observed-mean}{st.dev.}$, $Z = \frac{X-\mu}{\sigma}$, $Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$, $Z = \frac{\bar{p}-p}{\sqrt{\frac{p(1-p)}{n}}}$

CLT: 1. If n ≥ 30 then $\bar{X}$ is Normal with mean = μ and st. dev. = $\frac{\sigma}{\sqrt{n}}$

2. If $np \ge 5$ and $n(1-p) \ge 5$ then $\bar{p}$ is Normal with mean = p and st. dev. = $\sqrt{\frac{p(1-p)}{n}}$

Fact: If you start with a Normal Population, then $\bar{X}$ is *always* Normally Distributed with mean = μ and st. dev. = $\frac{\sigma}{\sqrt{n}}$

Confidence Intervals:

$\bar{x} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ $\bar{x} \pm t_{n-1,\alpha/2} \frac{s}{\sqrt{n}}$ $\bar{p} \pm Z_{\alpha/2}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$

Minimum Sample Size:

$n = (\frac{Z_{\alpha/2}\ \sigma}{E})^2$ $n = \frac{(Z_{\alpha/2})^2\ p^*(1-p^*)}{E^2}$

1. Bob and Evie are enthusiastic pet owners, but are unlucky when it comes to goldfish. Pet store owners know that the probability that a goldfish living for more than two weeks is 0.32. If Bob and Evie buy 10 goldfish, find the probability that 5 of them are still alive after two weeks. How many goldfish would you expect to still be alive and not belly-up after two weeks? Justify your calculations. （Hint: Use Binomial Distribution. Why? ）

2. An insurance company has determined that each week an average of 9 claims are filed in their Atlanta branch. What is the probability that during the next week
   a. exactly 7 claims will be filed?
   b. no claims will be filed?
   c. less than 4 claims will be filed in the following **3** weeks?

   (Hint: Use Poisson Distribution. Why?)

3. The distributor for Sun-missed oranges claims that the weights of these oranges follow a normal distribution with mean equal to 7 oz and standard deviation equal to .5 oz. (a) If one randomly selects an orange, what is the probability it weighs (i) more than 6 oz, (ii) between 7 and 8 oz, (iii) between 6.5 and 8.5 oz. (b) If one is told that the selected orange weighs more than 6 oz, what is the probability it weighs less than 7 oz?

4. Tom, Dick and Harry took the same class. The final scores for the class were normally distributed. Tom's score was 63, which was on the 59th percentile. Dick's score was 53, which was on the 38^{th} percentile. Harry's score was 85. On what percentile was Harry's score?

5. Suppose that a new type of brake light has been developed by GM. As a part of a product safety evaluation program, GM engineers wish to estimate the mean driver response time to the new brake light. (Response time is the length of time from the point when the brake is applied until the driver in the following car takes some corrective action). 51 drivers are selected at random, and the response time for each driver is recorded, yielding the following results: ¯x = 0.72, s^2 = 0.0936. Estimate the mean driver response time to the new brake light using a 95 % confidence interval.

6. A local bank reported to the federal government that its 5,246 savings accounts have a mean balance of $1000 and a standard deviation of $240. Government auditors have asked to randomly sample 64 of the bank's accounts in order to assess the reliability of the mean balance reported by the bank. The auditors say that they will certify the bank's report only if the sample mean balance is within $60 of the reported mean balance. What is the probability that the auditors will not certify the

bank's report, even if the mean balance really is \$1000? (Assume that the standard deviation reported by the bank is accurate.)

7. To receive a prestigious fellowship from a university the students have to pass a battery of tests. The first test is a verbal test. Suppose it is known from past experience that scores on the verbal test have a normal distribution with a mean score of 150 points and standard deviation of 30 points. A candidate will be considered for further screening only if his or her score on this test is 160 points or more. What fraction of the candidates will be eliminated at this stage? Give a full explanation.

8. A sample of 81 observations from a normally-distributed population produced a sample mean of 14.1 and a sample standard deviation of 2.6.

    A. Find a 95 % confidence interval for μ.

    B. Find a 99 % confidence interval for μ.

    C. What happens to the width of a confidence interval if the sample size is held fixed and the level of confidence is increased?

    D. Would your confidence interval be valid if the distribution of the original population was not normal? Explain.

9. A simple random sample of 400 people taken from a large population contains 40 smokers. Construct a 95% confidence interval for the percentage of smokers in the population.

10. Honda knows that gas mileages for Civics are normally distributed. If you talk to 10 Civic owners and get the data listed below, find a 90% CI for the true average gas mileage for a Civic.

    30, 38, 33, 34, 31, 35, 36, 36, 34, 33

    HINT: First compute the sample mean and standard deviation for the data before trying to construct the CI.